A Nonlinear Response Amplitude Operator for Complex Maritime Dynamical Systems, Wave Energy Converters and Offshore Applications

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ABSTRACT

A nonlinear response amplitude operator (NRAO) that is able to model a wide class of nonlinear systems and structures, usually found in maritime and offshore applications, is developed. Its functional scheme is of the Volterra-series type and its order is explicitly dictated from the order of nonlinearity of the actual dynamical system. In addition, the proposed method is not bounded by any amplitude or frequency constrains and it can compute the response of a nonlinear system even for multi-chromatic excitations that consist of modes with significantly, or not, different amplitudes and frequencies.

KEY WORDS: Nonlinear; Response Amplitude Operator; System Identification; Volterra; OWC; Wave Energy; Dynamical System;

INTRODUCTION

According to standard maritime engineering practice, in order to interpret the dynamic response of a floating body or an offshore structure with respect to the ambient wave excitations, a frequency transfer function, also termed as response amplitude operator (R.A.O.) is derived and used. The validity of this operator is based on the assumption that a linear system can be used to describe the relation between the excitation and the system’s response. A major advantage that comes with the analysis of dynamical systems in a linear context is that we can employ simple harmonic excitations, in order to obtain the required operator from a straightforward input-output relation and without any further computational effort.

However, in low or high dimensional nonlinear systems analysis, things can be far more complicated and the approximation functionals or their equivalent systemic structures, may not be able to successfully model the inherent nonlinearities of the actual system. In many cases, the test input used for the estimation of the nonlinear system’s elements, i.e., integral kernels or static polynomials, needs to be bi-chromatic, multi-chromatic or white Gaussian noise. Furthermore, the derivation of the resulting generalized transfer functions is based on tedious algorithms accompanied by expensive computational schemes that make continuous use of averaging techniques for any low and higher order auto and cross, input/output, correlations. Methods that employ harmonic inputs are usually fit for narrow band applications, whilst they are also quite constrained with respect to the excitation’s amplitude range. Furthermore, the target system needs to exhibit sufficiently smooth nonlinearities in the sense that the ratios between harmonics’ magnitude cannot vary significantly with the excitation’s amplitude or frequency.

The concept of a functional representation of a system of nonlinear differential equations through Volterra series was created on the basis of a generalization of power series solutions by Volterra (1959) and initially applied on nonlinear systems by Wiener (1942). The Volterra series approach, being a power series generalization, is related with several convergence issues, e.g., Palm and Poggio (1977). In consequence, Boyd et al. (1984) provided the conditions for the series to converge with the aid of various versions of the Gain Bound theorem, while Boyd and Chua (1985) derived, via the fading memory concept, suitable approximation theorems for a truncated Volterra series, defined on non-compact subsets of the input space, i.e. defined on the infinite line. Some very useful theoretical results with respect to classes of nonlinear dynamical systems that admit Volterra series modeling and identification were among others derived by Palm and Poggio (1978), Schetzen (1980), Rugh (1981) and Palm (1978). In the latter work in particular, equivalency conditions between different types of nonlinear systemic approximation schemes were derived. One of its main results, that is fully exploited in this work, is that all continuous-time dynamical systems can be approximated by the class of separable-kernel polynomial systems, a subclass of polynomial systems. It is noted that the term separable kernel corresponds to the ability to express a N-dimensional kernel as a N-product of one-dimensional (linear) kernels. However, even with such sound theoretical background on the capabilities of nonlinear system identification, the development of a widely applicable, generic approach seems to be lacking and we can only develop techniques that apply on specific characteristics of the nonlinear systems as is shown by Barrett (1963), Boyd and Chua (1984), Storer (1991) and Chatterjee and Vyas (2003, 2004), Lang and Billings (1997), Doyle et al. (2002), Ogunfumi (2007) and Giri and Bai (2010).

In an effort to overcome all of the above issues, a Volterra-series based nonlinear response amplitude operator (NRAO) is proposed. The order of the functional scheme is directly dependent on the order of...