Improvement of Energy Conservation in Particle Methods with Enhanced Schemes

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ABSTRACT
The MPS (Moving Particle Semi-implicit) method is a Lagrangian meshless method developed by Koshizuka and Oka (1996) to simulate incompressible fluid flows. However, its energy conservation properties have not been examined rigorously. In this paper, the effect of a few enhanced schemes in improving the energy conservation properties of MPS method is investigated by considering a set of appropriate Ocean Engineering related tests. It will be shown that an improved MPS benefiting from five enhanced schemes, referred to as, MPS-HS-HL-ECS-GC-DS with the Wendland kernel, provides relatively accurate energy conservation compared with the standard MPS. Furthermore, it tends to have the same level of performance in energy conservation with HMPS (Hamiltonian MPS; Suzuki et al., 2007) that theoretically should result in exact energy conservation.

KEY WORDS: MPS; Particle Method; Energy Conservation; Enhanced Schemes

INTRODUCTION
The MPS method was originally developed by Koshizuka and Oka (1996) as a Lagrangian meshless method for viscous incompressible fluid flow. In general, particle methods including the MPS method have a distinct advantage to reproduce the complicated violent flows with free surface owing to their Lagrangian tracking scheme without the advection term. However, particle methods with standard schemes have an inevitable shortcoming about unphysical pressure fluctuations that usually results in computational inaccuracy. Up to now, a number of enhanced schemes to suppress the pressure fluctuation have been developed (Gotoh et al., 2013). By applying the enhanced schemes to particle-based simulations, various challenging engineering problems including those in Ocean Engineering, such as breaking wave or wave impact have been widely studied. However, since MPS method is generally based on only the continuity and the Navier-Stokes equations, its energy conservation is not necessarily guaranteed, although theoretically, in case of constant temperature, the exact solutions in terms of mass and momentum conservations should satisfy energy conservation equation completely.

This paper aims at showing the improvements of energy conservation properties in the MPS method with a few enhanced schemes step by step. By presenting two sets of tests, namely, standing wave and solitary wave propagation, it will be verified that enhanced schemes mainly proposed for enhancement of stability and accuracy (in particular, reducing non-physical pressure fluctuations) also have good effects to reduce overestimation or underestimation of mechanical energy.

This paper also aims at illustrating the remarkable energy conservation property of the newest, the most enhanced MPS, referred to as MPS-HS-HL-ECS-GC-DS-WEND. The most enhanced MPS is proved to evaluate energy change quite accurately even compared with the HMPS (Hamiltonian MPS) that theoretically should lead to exact energy conservation. At last, the improved MPS can be said to satisfy not only continuity and momentum equations but also energy conservation equation up to an acceptable level.

NUMERICAL ANALYSIS METHOD

Standard MPS

In the MPS method, the motions of particles are calculated based on the following governing equations:

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} = 0$$

$$\frac{\rho}{Dt} \frac{D\mathbf{u}}{Dt} = - \nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g}$$

where $\mathbf{u}$ = particle velocity vector; $t$ = time; $\rho$ = fluid density; $p$ = particle pressure; $\mathbf{g}$ = gravitational acceleration vector and $\mu$ = dynamic viscosity.

Above governing equations are discretized by using differential operator models as follows:

$$\langle \nabla \phi \rangle_i = \frac{D_0}{n_0} \sum_{j \neq i} \left( \frac{\phi_j - \phi_i}{|r_j - r_i|} w(|r_j|) \right)$$

$$\langle \nabla^2 \phi \rangle_i = \frac{2D_0}{\lambda n_0} \sum_{j \neq i} \left( \phi_j - \phi_i \right) w(|r_j|)$$

where $\phi$ = an arbitrary scalar function; $D_0$ = number of scale dimensions; $r$ = coordinate vector of fluid particle ($r_j = r_j - r_i$); $w(r)$ = the kernel function and $n_0$ the constant particle number density. The