A Numerical Study of Nonlinear Wave Transformation and Harmonic Generation over a Submerged Step: The Effect of Step Height

Wei-Ting Chao 1  Chih-Chieh Young 2  Chao-Lung Ting 1
1. Department of Engineering Science and Ocean Engineering, National Taiwan University, Taipei, Taiwan China
2. Hydrotech Research Institute, National Taiwan University, Taipei, Taiwan China

ABSTRACT

The purpose of this study is to clarify the submerged step height effects on the harmonic generation using a vertically two-dimensional numerical wave flume. Eighteen experimental conditions were carried out, including different incident wave heights, periods and two step heights. The complicated spatial-temporal wave patterns were then analyzed by the two dimensional Fast Fourier transform (2DFFT) to separate incident/reflected waves. A new parameter that combines the conventional Ursell number Ur and the geometry factor \( d_s/h \) was further introduced to well predict the harmonic generation magnitude.

KEY WORDS: Harmonic generation, geometry factor, step height effect, numerical wave flume, Fast-Fourier transform.

1. INTRODUCTION

Nonlinear wave transformation and harmonic generation over a submerged step is a classical and important research topic in ocean engineering (Johnson et al., 1951). As waves propagate from the deep water region onto the submerged step, increased nonlinearity can lead to energy redistribution from the primary to higher frequencies, i.e. the so-called harmonic generation. In addition, each harmonic frequency would consist of free and bound waves. The phase mismatch between these two components determines the strength of the energy transfer (Madsen et al., 1993).

According to the past experimental and numerical studies, the geometry of a submerged step has an influence on the harmonic generation process (e.g. Dattatri et al., 1978; Patarapanich et al., 1989; Brossard et al., 2001; Christou et al., 2008; Brossard et al., 2009). A higher step can yield significant harmonic generation in which the harmonic amplitude is proportional to the incident wave height. By contrast, the effect of the step width is trivial. While the role of step geometry has been found and discussed qualitatively in the earlier works, a quantitative description between the step height and the harmonic magnitude has not yet been carefully addressed (see Li, 2013).

Accurate decomposition of incident and reflected wave components is helpful to better understand the physics of the energy transfer and harmonic generation in the shallow water region above a submerged step. Generally, the simple two-point method can be used to separate the incident and reflected waves (Goda and Suzuki, 1976). In addition to the two-point method, the four-wave-probe method can be further utilized to enhance the accuracy, especially for the highly nonlinear waves (Lin and Huang, 2004; Brossard et al., 2009). In comparison to the previous two approaches, the two dimensional fast Fourier Transform (2DFFT) provides more accuracy and efficiency for processing the wave signals (Kuo et al., 2009).

The purpose of the study is to investigate the effects of step height on harmonic generation. Eighteen experimental conditions were conducted in a numerical wave tank, including different incident amplitude (about 0.54 to 2.02 cm), periods (i.e. \( T = 0.8, 1.0 \) and \( 1.25 \) s), and two step heights (described by \( d_s/h = 0.24 \) and 0.55, where \( h \) is the still water depth; \( d_s \) is the water depth on the step or submergence depth of the step). For the preparatory study, the non-hydrostatic \( \sigma \) model (Young et al., 2007) was chosen and implemented with a higher-order scheme to obtain better computational accuracy and efficiency. The two dimensional fast Fourier Transform was utilized to analyze the numerical results. A new parameter that combines the Ursell number \( U_r \) and the geometry factor \( d_s/h \) was further introduced to well predict the magnitude of harmonic generation. The study results will serve as the basis for future laboratory conditions. In the following, Section 2 describes the numerical model and scheme. Section 3 presents numerical test conditions and data processing. Results and discussions are given in Section 4. Finally, short summary are drawn in Section 5.

2. Numerical model and scheme

2.1 Governing equations and boundary conditions

In this study, the non-hydrostatic \( \sigma \) model (Young et al., 2007) was utilized to conduct the numerical experiments. For describing free-surface waves, the governing equations are the unsteady, incompressible, Navier-Stokes equations (NSE). The Cartesian coordinate can be mapped into the \( \sigma \)-coordinate (as shown in Fig. 1) through the transformation \( t = t^* , \quad x = x^* , \quad \sigma = z - \eta (x^* , t^*) / H(x^* , t^*) \), where \( H(x^* , t^*) \) is the total water depth. Thus, the transformed continuity and momentum equations can be written as:

\[
\frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial \sigma} = 0
\]

(2-1)

\[
\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{w} \frac{\partial \bar{u}}{\partial \sigma} = \left( \frac{\partial \bar{P}}{\partial x} + \frac{\partial \bar{P}}{\partial \sigma} \right) + \nabla^2 \bar{u}
\]

(2-2)