Scaled Boundary Finite Element Method Analysis of Sloshing in Axi-symmetric Vessels

Jianbo Li, Jun Liu, Gao Lin
1. School of Hydraulic Engineering, Faculty of Infrastructure Engineering, Dalian University of Technology
2. State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology
Dalian, Liaoning Province, China.

ABSTRACT

This paper describes the development of a new efficient semi-analytical method, namely scaled boundary finite-element method (SBFEM) for the analysis of sloshing in axi-symmetric vessels based on multimodal analysis, and the governing Laplace equations and linear free surface boundary condition are also applied to the proposed model. A two-dimensional eigenvalue problem is obtained by using the scaled boundary finite element method for zero external excitation, which is solved through a variational (Garlerkin) formulation that uses three-node finite elements by only discretizing the boundary of the water surface and walls, then the dimension of the problem is reduced by one. Subsequently, based on an appropriate decomposition of the container-fluid motion, and considering the eigenmodes of the corresponding eigenvalue problem, an efficient methodology is proposed for externally-induced sloshing, through the calculation of the corresponding sloshing (or convective) masses. Numerical results are obtained for sloshing frequencies and masses in spherical vessels and a conical vessel, and the dynamic response of the sloshing is also obtained under different seismic waves. The numerical results are in very good comparison with other analytical or numerical solutions.

KEY WORDS: Scaled boundary finite element method, Liquid sloshing, Axi-symmetric vessels, Eigenvalue, Multimodal analysis.

INTRODUCTION

Liquid sloshing in a tank is considered as an important field of research in fluid mechanics. Liquid tanks have extensive applications in most aspects of engineering, ranging from fuel tanks in flight vehicles, super tankers, marine systems as well as seismic studies of structures under violent hydrodynamic pressure due to sloshing. If a liquid tank is externally excited at or near a natural frequency of sloshing, the resonance phenomenon occurs, which may bring out violent hydrodynamic pressure on the tank walls and result in subsequently the failure of the whole system. Meanwhile, violent motion of the tank may lead to such a problem. Hence, measurement of hydrodynamic pressure and investigation has engaged many researchers during several decades and are still under scrutiny. There has been many research works in the literature dealing with modeling of sloshing to estimate sloshing frequencies, modes shapes, and hydrodynamic forces under different excitation for arbitrary geometry containers, which can be performed using experimental models, either analytical approaches or computational techniques or (Ibrahim et al., 2001; Ibrahim, 2005; Faltinsen and Timokha, 2009). With the rapid development of the computer performance over half a century, computational methods have become robust tool for many field analyses of sloshing problems. Among them, the finite element method (Mitra et al., 2008; Kumar and Sinhamahapatra, 2014), boundary element method (Firouz-Abadi and Borhan-Panah, 2013; Kolaei et al., 2015), and finite difference method (Buldakov, 2014) et al. have become many successful uses in numerical analysis of sloshing during the last three decades. It is well accepted that, among those methods, the FEM is undoubtedly the dominant one for modelling sloshing problems at present, because of its powerful capability of simulating a large variety of problems with complex structural geometries and various boundary conditions. However, the standard FEM yields comparatively poor results when applied to the liquid container whose domain contains re-entrant corners (such as baffles in containers), owing to the singular nature of the solution. In order to circumvent this difficulty, the FEM mesh is to refine locally in the region of the singularity, which will lead to large computational effort, or to introduce singular elements, which incorporates the asymptotic singular stress functions. On the other hand, the BEM is an attractive technique for solving the problems, since only the boundary is discretized which results in a reduction of the spatial dimension by one. However, distinct disadvantages are associated with BEM, i.e., a fundamental solution is required and singular integrals exist. In the literatures, most studies have focused on vertical-cylindrical tanks. On the other hand, containers in other geometries, such as spherical containers or other axi-symmetric vessels, have received much less attention. Those vessels have significant industrial applications in chemical plants or refineries, and are employed as storage vessels for liquefied petroleum gas (LPG), liquid propane, propylene and LNG.

In this paper, a novel semi-analytical method called scaled boundary finite element method (SBFEM) has been presented as an attractive