Asymptotic Expansion of the Transient Capillary-Viscosity Green Function

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ABSTRACT

The asymptotic expansion of the transient capillary-viscosity green function at large distance \( r \) and large time \( t \) is considered in this paper. Difficulties arise when \( r / t \) approaches the minimum speed of capillary-gravity waves \( v_0 \). A cubic transform of the variable of integration is used to overcome the problem and uniformly valid asymptotic approximations in terms of the Airy functions and its derivatives are presented which span three separate domains for \( r / t \) greater than, near, and less than \( v_0 \). Calculations are performed to illustrate the utility of the asymptotic results.

KEY WORDS: Capillary gravity waves; uniform asymptotic approximations; Airy functions.

INTRODUCTION

The solution to the classical Cauchy-Poisson problem of water waves generated by an impulsive disturbance under the pure-gravity effect (Lamb 1932) presents a perplexing peculiarity - the surface elevation in a region approaching to the impulsive disturbance (an initial elevation concentrated along a line of surface in 2D case) is found to diminish continuously in length and to increase continuously in height without limit.

In 3D case, the same situation has been revealed for the steady translating source by Ursell (1960) and for the transient source by Clement (1998) and for the oscillatory translating source by Chen and Wu (2001). This peculiar property of pure gravity waves hinders the numerical development to solve the boundary-value problem associated with a floating body in which the space integral over body’s surface as well as the time-convolution integral are difficult to be accurately obtained.

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The surface tension is considered to be significant only for rather short waves such as ripples and commonly ignored in describing water waves around large floating bodies. However, the theory of gravity waves may yield these highly oscillatory waves of very short length which are manifestly non-physical, it is expected that the surface tension plays an important role to eliminate the singular behavior of these short waves. In fact, the capillary gravity waves have been studied since Kelvin (1871) as summarized in Wehausen & Laitone (1960).

Chen (2002) gave a updated analysis on the steady ship waves including the effect of surface tension. It is shown that the introduction of surface tension in the formulation of ship waves eliminates the singularity of ship waves in the region near the track of the source point at the free surface.

Chen & Duan (2003) considered capillary-gravity waves due to an impulsive disturbance and performed asymptotic analysis on the transient waves due to an impulsive source, especially, at large time as well as in the region near the disturbance. Dependent on the wavenumbers associated with the level of the effect due to surface tension, the oscillations were grouped as gravity-dominant waves and capillary-dominant waves by Chen & Duan (2003). It is confirmed that the gravity dominant waves represent faithfully the physical properties of free-surface waves.

In the framework of linear Stokes equation, Chen et al. (2006a) and Chen et al. (2006b) considered the potential flow induced by a point impulsive force and source, the combined effect of fluid viscosity and surface tension on the potential function at the water surface was studied and the transient capillary-viscosity green function was given by Chen et al. (2006b), it is shown that the wave form of gravity-dominant oscillations is mostly modified by the surface tension while the wave amplitude of capillary-dominant oscillations is mostly reduced by the fluid viscosity.

Let \( v_0 \) be the minimum value of propagation velocity of capillary-gravity waves. Chen et al. (2006b) used the method of steepest descents to obtain the asymptotic expansion of the transient capillary-viscosity green function for \( v > v_0 \) and \( v < v_0 \) respectively. Both approximations fail for the transition region \( v = v_0 \) which corresponds to a coalescent pair of saddle points in the Fourier integrals. The third approximation using Airy functions can describe the transition for \( v = v_0 \) between the oscillatory motion for \( v > v_0 \) and the exponential small motion for \( v < v_0 \). However these local approximations are limited in their domains of validity and overlap among them cannot be achieved uniformly for large time.