Risk Assessment in Sloshing: How to Deal with Multidimensional Datasets.

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ABSTRACT

In the liquefied natural gas (LNG) shipping industry, the phenomenon of sloshing can lead to the occurrence of high pressures on the tanks walls of the vessel. The issue of modelling or estimating the probability of the simultaneous occurrence of such extremal pressures is now crucial from the risk assessment point of view since the probability of failure of the system depends of the area of the sloshing impact. Multivariate heavy-tailed distributions are considered, with sloshing pressures investigated by means of small-scale replica tanks instrumented with \( d \geq 1 \) sensors. When attempting to fit such nonparametric statistical models, one naturally faces computational issues inherent in the phenomenon of dimensionality. The primary purpose of this article is to overcome this barrier by introducing a novel methodology. For \( d \)-dimensional heavy-tailed distributions, the structure of extremal dependence is entirely characterised by the angular measure, a positive measure on the intersection of a sphere with the positive orthant in \( \mathbb{R}^d \). As \( d \) increases, the mutual extremal dependence between variables becomes difficult to assess. Based on a spectral clustering approach, we show here how a low dimensional approximation to the angular measure may be found.

KEYWORDS: Sloshing, multivariate heavy-tail distribution, asymptotic dependence, spectral clustering.

INDUSTRIAL CONTEXT

The membrane cargo containment (CCS) system is the most widely used technology to convey LNG. Although this technology is highly reliable, it can be susceptible to sloshing: waves impact of LNG may induce high pressures on the tank walls and potentially damage the CCS. The phenomenon is being studied by GTT experimentally on instrumented small-scale replica tanks (1 : 40 scale). The tanks, partially filled with water, are shaken by a jack system to reproduce the motion of the ship after relevant scaling and induce the occurrence of sloshing. Pressures are recorded by numerous sensors arranged in arrays in the impact areas.

The phenomenon to be analysed here is described by a series of pressure measurements, and in particular by the existence of peak values corresponding to pressures created by sloshing impacts. Hence heavy-tail modelling is relevant in this context for a conservative risk assessment insofar as it generally does not underestimate the importance of extreme values. Heavy-tailed distributions are also used for risk assessment in many other fields such as in finance (Rachev et al., 2005), insurance (Mikosch, 1997) or for modelling natural hazards (refer to Wadsworth and Tawn (2012) or Coles and Walsh (1994)). Since a damage may occur when the liquefied gas gives a heavy impact to a large area of the tank, it is crucial to assess accurately the probability of occurrences of high pressures at several sensor locations during a given impact. This paper considers the problem of estimating this key information in a nonparametric manner.

Assuming that sloshing data are derived from a multidimensional heavy-tail model, then when expressed in polar coordinates the radial part is asymptotically distributed as a standard Pareto variable and independent of the angular component. The (asymptotic) distribution of the angular component is referred to as the angular measure on the intersection of the unit sphere with the positive orthant of \( \mathbb{R}^d \). The extremal dependence between all \( d \) sensors in the tank (or all the sensors in a specific area of the tank) is completely characterised by the angular measure. When the tank is instrumented with \( d \) sensors, the angular measure can be decomposed into a mixture of up to \( 2^d - 1 \) sub-angular measures, with dimensions ranging from 0 to \( d - 1 \). Hence, any direct method for estimating the angular measure would suffer from the curse of dimensionality. Hence, we seek a segmentation of the collection of sensors into \( l \) groups such that for one group: (i) the measurements collected are mutually dependent in the extremes, (ii) these measurements are mutually independent in the extremes from those of the other groups. Ideally, the cardinalities of the groups should be small with respect to \( d - 1 \) and \( l \) small with respect to \( 2^d - 1 \), so that estimation of the angular measure becomes tractable. For this purpose, we introduce here a novel methodology grouping the sensors into clusters satisfying assumptions (i) and (ii). This method is based on a spectral clustering algorithm (von Luxburg, 2007), tuned to detect asymptotic dependences and independences. Ultimately, by conditioning upon the membership in each cluster, the asymptotic distribution of the data can be simulated and the corresponding risk of failure assessed.

SLOSHING DATA

We start with a description of the sloshing data on which the subsequent statistical analysis relies, and then briefly review the basic concepts of heavy-tail analysis.

Data set

The data set provided by GTT corresponds to a tank located in the front of the vessel, in the high filling configuration where the tanks are nearly full of liquefied gas. The tank is instrumented with a collection