Application of Hamilton-Dirichlet’s Principle to Analysis of Hydroelastic Behavior of an Elastic Floating Plate of Arbitrary Plan Geometry in Waves

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ABSTRACT

In this study, in order to calculate wave-induced responses of an elastic pontoon type floating plate of arbitrary plan geometry in waves, a new method is proposed which uses “Modified Hamilton-Dirichlet’s Principle 2” considering wave radiation condition and the eigenfunction expansion method for fluid motion. The velocity potentials in fluid regions with and without the plate are expanded by eigenfunctions in vertical mode which satisfy the governing equations and free-surface conditions, taking into account the presence of the plate in the same manner as Kim and Ertekin. In this method, “Modified Hamilton-Dirichlet’s Principle 2” is finally reduced to a variational equation which corresponds to boundary conditions on the plate’s edge. The formulation of the proposed method is applicable for the floating plate of arbitrary plan geometry.

Calculated results of two types of rectangular and L-shaped floating plate in open sea are compared with experimental results. Good agreement is found between computed and experimental results.

KEY WORDS: Pontoon-type VLFS, Wave-induced elastic motion, Variational principle, Radiation condition, Eigenfunction expansion method

INTRODUCTION

Pontoon-type VLFS is one of the typical structural types of very large floating structures (VLFS). Various numerical methods have been proposed to predict the hydroelastic response of this structure in waves (Watanabe (2004); Chen (2006)). These methods are classified into the modal expansion method and the direct method. These analyses are carried out in the frequency domain or in the time domain. Most analysis so far dealt with VLFS of rectangular planform. There are also methods using the finite element method for the structure in order to analyze actual complicated floating structure (Seto et al. (1998); Utsumomiya et al. (2002)).

In relation to the elastic response of Pontoon-type VLFS in waves, Isshiki and Nagata (2001) derived four kinds of variational principles related to elastic motions of such a floating plate and made clear the mutual relationship of them. First, Hamilton-Kelvin’s Principle, which is one of these variational principles for motions of an elastic plate in waves, is obtained by combining Hamilton’s Principle related to motions of a plate and Kelvin’s Principle related to fluid motion. In this principle, kinematic conditions in motions of plate and water are constraint conditions, and mechanical conditions in motions of plate and water are natural conditions. In this variational principle, arguments for fluid motion are the velocity vector of water and the water elevation. The use of velocity potential is more convenient than that of velocity vector in solving the fluid motion in water wave. Therefore, Hamilton-Dirichlet’s Principle is obtained by transforming the velocity vector in Hamilton-Kelvin’s Principle into the velocity potential. In this principle, kinematic condition in motions of plate and mechanical condition in motion of water are constraint conditions. Mechanical condition in motions of plate and kinematic condition in motion of water are also natural condition. Further, if the solution of fluid motion is obtained by some method, Modified Hamilton-Dirichlet’s Principle-1 is obtained by constraining the natural condition on motions of water in Hamilton-Dirichlet’s Principle. By eliminating the vertical displacement of the plate in Hamilton-Dirichlet’s Principle by using kinematic conditions between plate and water, Modified Hamilton-Dirichlet’s Principle-2 in which motions of plate and water are expressed only in velocity potential is also obtained. However, in these variational principles, wave radiation condition is not included and the horizontal velocity in the infinity is designated.

In order to calculate hydroelastic responses of a pontoon-type structure of arbitrary plan geometry in waves, authors proposed an analytical method which used the “Modified Hamilton-Dirichlet’s Principle-2” and the eigenfunction expansion method for fluid motion (Niizato et al.2009). The argument function in Modified Hamilton-Dirichlet’s Principle-2 is only velocity potential and the deflection of plate is not included as the argument function. The velocity potentials in fluid region under the plate are expanded by eigenfunctions in vertical mode which satisfy the governing equations, taking into account the presence of the elastic plate in the same manner as Kim & Ertekin (1998,1999) for VLFS of a rectangular planform. Thereby, Modified Hamilton-Dirichlet’s Principle-2 is finally transformed in a variational equation which satisfies the boundary conditions at the edge of the plate, that is, the bending moment and shear force should vanish at the