Boussinesq Modeling of Undertow Profiles

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ABSTRACT

Though Boussinesq-type wave equations are widely used in the nearshore wave field, it is still a challenge for these kinds of equations to describe the velocity profiles along water column after wave breaking occur. For example undertow, plays a key role in generating sandbars, is quite difficult to be simulated by a Boussinesq-type model (usually used in a phase-resolving beach evolution model). The ad-hoc method developed by Lynett (wave breaking velocity effects in depth-integrated models. Coastal Engineering, 2006, 53(4):325-333) seems greatly improve the performance of Boussinesq-type equations for undertow calculation. However this method is a post-processing modification skill, the accurate prediction of undertow profiles greatly depends on the accuracy of predicted wave elevation and depth-averaged velocities before modification. Hence Boussinesq-type equations with fully nonlinear characteristics instead of those with only weak nonlinearity should be used to simulate breaking waves, as proved by many researches that the former could give better results than the later in wave breaking zone. In the present paper, a new set of fully nonlinear Boussinesq equations and the above mentioned ad-hoc method are combined to form a wave breaking model, aiming to simulate the undertow profiles. The model is first validated by simulating regular waves breaking on a plane beach, then the undertow profiles are calculated, numerical results are compared against three available data sets and the performance of wave breaking model before and after modification is detailed discussed.

KEY WORDS: Wave breaking; Boussinesq; undertow.

INTRODUCTION

Cross-shore time-averaged velocity below wave trough, or undertow, plays an important role in various coastal engineering problems, especially in the prediction of cross-shore sediment transport rate, which is the most determinant factor in a sediment transport model. In the most state-of-the-art processed-based morphodynamic models, Boussinesq-type wave modules are usually adopted as a wave driver to force the movement of the sediment due to its outstanding performance in the nearshore region (e.g., Rakha et al., 1997; Karambas and Koutitas, 2002; Long et al., 2006; Fang et al., 2010). However, these kinds of wave modules belong to depth-integrated models with a typical assumption that the vertical profile of velocity can be represented by a polynomial, wherein the order of the polynomial is proportional to the accuracy of the resulting model. Implicit with this velocity profile, and often a direct inviscid assumption, is a lack of ability to simulate turbulence. Employing some technical skills like eddy viscosity wave breaking mechanism, porous beach assumption and empirical quadratic bottom friction, Boussinesq-type model still could be used in the nearshore region where extremely complicated wave motion occurs. And wave height, setup up, even runup could be reproduced well by carefully tuning the parameters involved in the model (e.g., Kennedy et al., 1999). For velocity profiles, however, Boussinesq-type model has distinguished performance before and after wave breaking. For non-breaking waves, the vertical profile of velocity is predicted very well, even for strongly nonlinear waves (e.g., Wei et al., 1995; Ryu et al., 2003), provided the wave is not in deep water. While the vertical profile of velocity under breaking wave is poorly predicted (e.g., Lynett, 2006; results in the present paper). This brings the difficulties into transport calculation as it is very sensitive to accurate representation of the vertical velocity profiles. To work around this obstacle within the Boussinesq framework, researchers have developed some new Boussinesq models with the ability of describing rotational wave motion after wave breaking (e.g., Veeramony and Svendsen, 2000; Musumeci et al., 2005). These new types of models yield very good agreements when compared to the undertow data. In these models, however, vorticity transport equation is solved semi-analytically or totally numerically, leading to a rather complex implementation as the generation and motion of the vortex should be captured.

Lynett (2006a) proposed an alternative method to calculate the undertow profiles in a two-layer Boussinesq model, this method is simple and the computed undertow profiles are in well agreements with the data of Cox et al. (1995), Ting and Kirby (1994). In this method, only vertical profile of velocity is modified and this modification does not change the calculation of other hydrodynamics, as named a post-processing method. And the accurate prediction of wave height (setup etc.) is used as a basic assumption. The Boussinesq-type equations with fully nonlinearity characteristics seem to be the optimal choice for this purpose as proved by many researchers that Boussinesq-type model with fully nonlinearity could give better prediction than those only has

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