A Finite Element Method for Calculating the Applied Top Tension and Static Configurations of Extensible Marine Riser with Specified Total Arc-Length

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ABSTRACT

A variational formulation of an extensible marine riser is formulated based on the work-energy principle. The total unstretched arc-length of marine riser is specified while the top tension is not yet exactly known at the equilibrium position. A Lagrange multiplier is introduced in order to impose the constraint condition, which is the specified total arc-length of the riser. The system unknowns are composed of the nodal degrees of freedom and the Lagrange multiplier. The system of nonlinear finite element equations is derived based on the finite element procedure. The numerical solutions of the nonlinear system are obtained by the iterative method. The results show that the Lagrange multiplier is identified as the value of top tension adjusting the specified top tension in order to maintain the marine riser in equilibrium condition.

KEY WORDS: Extensible Marine Riser; Applied Top Tension; Constraint Condition; Lagrange Multipliers; Static Configurations; Finite Element Method.

INTRODUCTION

The marine risers have been employed in offshore resource exploration form the 1950s. They are used to contain fluids for well control and to transport hydrocarbons from the wellhead to the platform. In deep water operation, the riser behaves as a flexible structure which experiences large displacement. The mathematical model for marine risers with large deflection analysis had been developed over the past several years.

In literature, there are many research works which deal with the large displacement of the risers, for example, Felippa and Chung (1981), McNamara et al. (1986), Bermitas and Kokarakis (1988), Chung et al. (1994, 1996), Moe and Amtsne (2001), and Chai and Varyani (2006). Most of them used the arc-length of the riser as the independent variable to define the centroidal line of the riser. However, in most cases, the top end of the riser can slip through the slip joint. Consequently, the total stretched arc-length of the riser measured from the seabed to the slip joint may not be known until the equilibrium configuration is evaluated. Therefore, the use of the unstretched arc-length to be the independent variable may not convenient to set up the boundary condition at the slip joint.

In order to reduce the complexity of the problem discussed above, the vertical distance is used as the independent variable instead of the unstretched arc-length (Huang and Chucheepsakul, 1985). This technique eliminates a number of iterations that are required to adjust the total unstretched arc-length until the boundary conditions at the top end are satisfied. Moreover, in finite element analysis, the discretization of the riser elements along the arc-length requires more nodal variables than the discretization along the sea depth (Chucheepsakul et al., 2003). The numerical solutions, that are determined by using the vertical distance as the independent variables, can be found in many research works such as Huang and Kang (1991), Chuoeepsakul et al. (1999), Athisakul et al. (2002), Monprapussorn et al. (2004), Kaewunruen et al. (2005), Monprapussorn et al. (2007), Athisakul et al. (2008).

For the case of the applied top tension is specified, the discretization along the sea depth is suitable as the discussion above. The stretched arc-length and unstretched arc-length can be easily found by direct integration along the vertical coordinate of the riser configurations. On the contrary, if the unstretched arc-length is specified, the applied top tension will be adjusted to satisfy the equilibrium and boundary conditions. A trial error in estimating the proper value of the applied top tension may be used. However, this constraint problem can be solved directly by using the Lagrange multiplier method.

The purpose of this paper is to present the finite element method for large displacement and large deformation analysis of marine riser with a constraint condition (a specified total unstretched arc-length). The model formulation is developed by using the variational approach. The strain energy due to bending, axial stretching and virtual work done by hydrostatic pressure and other external forces are involved in the variational model. A Lagrange multiplier is introduced in order to impose the constraint condition. The numerical examples are provided to explain the physical meaning of the Lagrange multiplier. The relations between the top tension and the unstretched arc-length in different water depth and static offset are investigated herein.