Numerical simulation of breaking wave with Level Set method

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ABSTRACT

Breaking wave has a tremendous destructive power to the vertical breakwaters, and needs further study with numerical model. Based on unstructured grid, a two-dimensional numerical wave flume model is presented in this paper, which is a combination of three-step finite element method, Level Set method and large eddy simulation. The impact course of breaking wave is simulated with this model, and the numerical results of wave force and flow field have good agreement with the Code of Sea Port(China) and PIV experimental data. This shows that this Level Set-FEM combined numerical model can simulate breaking wave well.

KEY WORDS: Level Set; finite element method; breaking wave; PIV.

INTRODUCTION

When vertical breakwaters locate on mounds, the wave may break on the top of mounds, and the breaking wave occurs. Compared with standing wave, breaking wave has a more tremendous destructive power to the vertical breakwaters. But the mechanism of breaking wave hasn’t found out until now, and need to be studied systematically with numerical method in which the free surface of wave can be captured accurately. Conventional VOF method, which is based on the finite difference method and applied with structured grids, is the most popular method to track free flow surface. But the boundary shapes of vertical breakwaters with their mounds are usually complex. If the usual rectangular grids are employed, the boundary shapes of the structures are difficult to be described in the numerical model and the finite difference method may lead to inaccurate results. Level Set method is a new interface-capturing method which was presented by Osher and Sethian (1988) and has been widely used in many fields these years. By advecting Level set function with local velocity field, the interface position can be captured. Compared with VOF method, one of the advantages of Level Set method is that Level set method is much easier to be applied with finite element method which has a good performance for problems with complex boundary configurations.

In this paper, a two-dimensional numerical wave flume model is proposed, which is a combination of three-step Taylor-Galerkin finite element method, large eddy simulation with the Smagorinsky sub-grid model and Level Set method. To avoid the mass imbalance of fluids due to the imperfect Eulerian velocity field in the numerical model, a global mass conservation scheme is also employed. By using this model, the breaking wave impacting on a vertical breakwater is investigated, and the numerical results of wave force and flow field have good agreement with the code of sea port (China) and PIV experimental data. Good performance of this numerical wave flume model is demonstrated.

GOVERNING EQUATIONS AND DISCRETIZATION

The basic equations are the following continuity equation and two-dimensional Navier-Stokes equations for incompressible, viscous fluid.

\[ \frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu_1 \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right) + f_i \]  

where, \( u \) is the velocity; \( \rho \) is the density of fluid; \( p \) is the pressure and \( f \) is the body force; \( \nu_1 \) is the effective viscosity coefficient and \( \nu_2 \) is the turbulent viscosity. In this study, the Smagorinsky SGS turbulence model is employed, so

\[ \nu_1 = (c \Delta)^4 \left[ \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j} \right] \]  

where \( c \) is the Smagorinsky constant ranging from 0.1 to 0.2, and \( \Delta \) is the characteristic length of the cell.

The governing equations are discretized with three-step finite element method (Jiang and Kawahara, 1993) which has third-order accuracy. In this method, time-splitting idea is employed and the discretization of Navier-Stokes equations in time is carried out by Taylor series expansion, while the spatial discretization is obtained by the standard Galerkin method. It is notable that the same order of interpolation is used for velocity and pressure. The formula of three-step finite element method is as follows,

\[ \frac{u_i^{n+1} - u_i^{n}}{\Delta t} - u_i^{n} \frac{\partial u_i^{n}}{\partial x_i} - \frac{1}{\rho} \frac{\partial p^{n}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu_1 \left( \frac{\partial u_i^{n}}{\partial x_j} + \frac{\partial u_j^{n}}{\partial x_i} \right) \right) + f_i^{n} \]