Application Research of Homotopy Analysis Method in Nonlinear Water Wave Equations

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ABSTRACT: Water wave problems show strong nonlinear characteristics which lead to some difficulties in the study. Among the research methods in point, Homotopy analysis method (HAM) has many advantages since it doesn’t rely on a special small parameter. So we use HAM to solve water wave problems with strong nonlinearity, where the main work includes basic scheme based on HAM and comparison with a traditional method, i.e. a modified double curvilinear method. Based on numerical simulations it is found the method proposed in this paper is effective.

KEY WORDS: homotopy analysis method, water wave equation, nonlinear characteristic

1. INTRODUCTION

A water wave equation has strong nonlinear characteristic which is just the main difficulty especially in analyzing stability of offshore structure. Before analyzing the stability, it’s important to determine equation’s solution quickly. Among the ordinary methods for such a topic are numerical method, analytic method, experimental methods, etc. Nowadays as the computing technology has been highly developed, numerical simulation is the main technique but it will be invalid when singularity or multiple solutions exist. Analytic methods, such as widely-used perturbation methods, can overcome such problems above, but usually its effectiveness is based on special conditions such as the existence of a special small parameter or variable and only for weak nonlinear problems is it valid. For strong nonlinear problems we usually cannot get convergent solutions with the aforementioned methods.

Homotopy analysis method (HAM) (Liao, 2004) has many advantages in nonlinear problems. For instance, it can control convergence region and adjust convergence rate. This novel method doesn’t rely on a special small parameter (Liao, 1999) and it is proved to be desired in many aspects. For strong nonlinear problems we usually cannot get convergent solutions with the aforementioned methods.

2. A GENERAL NONLINEAR WAVE EQUATION

Consider a general nonlinear wave equation (Zhou, etc. 2006),

$$\frac{\partial^2 u(x,t)}{\partial t^2} - a_1 \frac{\partial^2 u(x,t)}{\partial x^2} + a_2 \frac{\partial u(x,t)}{\partial t} + a_3 u(x,t) + a_4 u^3(x,t) = 0 \quad (1)$$

where \(u(x,t)\) is the wave vector, the coefficient \(a_i (i = 1, 2, 3, 4)\) is dependent of location and time. When these coefficients are modified, Eq.1 can be reduced to some famous equations such as Sincordon Equation, Klein-Cordon Equation (Georgiev, etc, 2006), Landau-Ginzburg Equation (Rosier, etc. 2009), and so on. As to now, people have proposed many methods to solve exact solutions of equations like Eq.1, for example, coefficient equilibrium method, double curvilinear method, Jacobi elliptical function method, etc. Due to holding effectivenss and simplification, modified double curvilinear method (MDCM) is often used to solve exact solution of Eq.1. Besides, some approximate solution may be enough to some problems. Thus we try to use HAM to obtain approximate solution of Eq.1, then compare its result with MDCM.

3. WAVE EQUATION’S EXACT SOLUTION BASED ON MDCM

3.1 BASIC IDEA OF MDCM

Exact solution based on MDCM is from the idea of perturbation. We use a solvable Riccati equation to perturb Eq.1, then construct the equation’s explicit solution by Riccati equation. If a wave equation can be written as the Riccati equation (Tang etc., 2007), as follows

$$\frac{\partial v}{\partial \xi} = b + v^2 \quad (2)$$

where \(v\) is a function with respect with an argument \(\xi\). If \(\xi = x - ct\), where \(c\) is the wave speed, then Eq.2’s solution is decided by the sign of parameter \(b\). Concretely, 

(a) when \(b < 0\), Eq.2 has the following solitary solution

$$v = -\sqrt{-b} \tanh(\sqrt{-b} \xi) \quad (3)$$

$$v = -\sqrt{-b} \coth(\sqrt{-b} \xi) \quad (4)$$

where Eq.3 denotes stable solitary wave and Eq.4 means emanative...