On the Effect of Compressibility on the Impact of a Falling Jet

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ABSTRACT

At the first World Sloshing Dynamics Symposium that took place during the Nineteenth (2009) International Offshore and Polar Engineering (ISOPE) Conference in Osaka, Japan, it was made clear that simplified academic problems have an important role to play in the understanding of liquid impacts.

The problem of the impact of a mass of liquid on a solid structure is considered. First the steady two-dimensional and irrotational flow of an inviscid and incompressible fluid falling from a vertical pipe, hitting a horizontal plate and flowing sideways, is considered. A parametric study shows that the flow can either leave the pipe tangentially or detach from the edge of the pipe. Two dimensionless numbers come into play: the Froude number and the aspect ratio between the falling altitude and the pipe width. When the flow leaves tangentially, it can either be diverted immediately by the plate or experience squeezing before being diverted. The profile of the pressure exerted on the plate is computed and discussed. Then the same problem is revisited with the inclusion of compressibility effects, both for the falling liquid and for the gas surrounding it. An additional dimensionless number comes into play, namely the Mach number.

Finally, a discussion on the differences between the incompressible and compressible cases is provided.

KEY WORDS: Jet; liquid impact; dimensionless number; free-surface flow

INTRODUCTION

The problem of a liquid jet impacting on a wall is a classical one, with very practical applications, in particular in sloshing. Indeed the impact of a fluid on a solid boundary often occurs as a mass of liquid pushing the gas around it and hitting the structure ahead. It is a complex fluid mechanics problem as suggested by some of its features, such as: the nonlinearity of its free surface; the possible presence and importance of the compressibility effects, when gas is trapped by the liquid; the role of the elasticity of its structure. A description of all phenomena which can take place when a liquid jet hits a structure was provided by Braeunig et al. (2009) at ISOPE 2009. It was emphasized that simplified or academic impact conditions can be quite useful in understanding various phenomena.

The simpler problem of an infinite falling jet has already led to several papers, at least in the context of incompressible flows. The main motivation was the study of a long bubble rising through an infinite plane vertical tube of liquid. This problem can be actually viewed – if one uses a co-ordinate system attached to the bubble – as a liquid falling around a bubble, instead of the bubble rising in the liquid. The problem of an incompressible falling jet impacting on a horizontal plate was solved recently by Christodoulides and Dias (2010). The main results of that paper will be reviewed. Then we provide a better understanding of the compressible flow impacting on a solid plate. Since the geometry is exactly the same for the incompressible and the compressible cases, the geometric description is provided only once.

A stream of liquid flows down and out of the bottom of a long two-dimensional vertically-sided pipe of half-width $W$. The downwardly directed flow meets a horizontal plate of infinite extent set at a distance $H$ below the bottom end of the pipe. The flow splits into two jets on each side of the pipe following a path along the horizontal plate. The general solution depends on the ratio $H/W$, on the dimensionless Froude number

$$F = \frac{U}{\sqrt{gW}}, \tag{1}$$

where $g$ is the acceleration due to gravity and $U$ the velocity of the fluid far inside the pipe, and on the Mach number

$$M = \frac{U}{c}, \tag{2}$$

where $c$ is the speed of sound in the liquid. The incompressible case corresponds to the limit $M = 0$. A major difference between the incompressible and compressible cases is that the gas motion is modeled as well in the compressible case. Therefore there is an additional dimensionless number, which is the density ratio between the