DRBEM Solution of Extended Mild-Slope Equation for Waves around a Circular Island on a Polynomial Shoal

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ABSTRACT
In this paper, the extended mild-slope equation was adopted to be the governing equation and solved by dual reciprocity boundary element method. This model improves the accuracy of former papers caused by neglecting the curvature and slope-squared terms in their models. Due to the former works focusing on the long wave, the extended terms become very small and were omitted. Recently, the extended terms was considered that cannot be omitted even the long wave. A series of numerical experiments were conducted including conical island, Homma’s island. The Homma’s island was the case studies and the results were compared with the analytical results derived by Homma (1950) and/or Jonsson et al. (1976). Excellent agreements were obtained.

KEY WORDS: Extended mild-slope equation; dual reciprocity boundary element method; Homma’s island; conical island; wave model.

INTRODUCTION
To simulate linear wave propagation from deep to shallow water, the use of the mild-slope equation derived by Berkhoff (1972) can be a proper and valid method. The classical mild-slope equation is based on the assumption of a slowing varying bathymetry, linearizing the scattering of surface waves on variable water depth by approximating the vertical structure of motion and averaging over depth. Thus, the dimension of the problem can be reduced by one. Due to the assumption of $|\nabla h|/kh << 1$ on mild-slope equation, the higher-order bottom effect terms are neglected during the original derivation procedure of Berkhoff. Motivated in part by the significant engineering applications, much of the relevant existing literature has concentrated on rapidly-varying or steep topography. Booij (1983) compared the numerical results of mild-slope equation with the finite element model results in terms of the reflection coefficients for the case of monochromatic wave propagation over a plane slope. He made a conclusion that the mild-slope equation is sufficiently accurate up to a bottom slope of 1:3. However, it has been pointed out in a lot of investigations that the classical mild-slope equation fails to produce adequate approximations for certain type of bathymetry, such as off-shore reef or bars. Numerous works have been conducted to improve the applicability of the mild-slope equation for rapidly and relatively steep bathymetry. Chamberlain and Porter (1995) proposed an extended refraction-diffraction equation (EMSE), which contains the curvature and the slope-squared terms by utilizing the variation principle and the Galerkin method. It is to note that the most important part of Chamberlain and Porter’s improvement was in retaining a term involving second derivatives of the quiescent depth $h$. In addition, Porter and Staziker (1995) proposed a modified mild-slope equation (MMSE). The appearance of $v^2h$, where $v$ is the gradient operator, in the MMSE implies that matching conditions must be applied to the dependent variable when $v h$ is discontinuous. It means that the classical mild-slope equation fails to preserve mass conservation due to discontinuity in the bed slope and leads to the less accurate prediction. A similar work was conducted by Chandrasekera and Cheung (1997). They derived an alternative mild-slope equation including the bottom curvature and slope-squared terms. The equation applied the hybrid element method to simulate wave reflection from a ripple bed and wave transformation over a circular shoal. According to the studies stated above, almost all of them indicated that the higher-order bottom terms including the curvature and the slope-squared terms cannot be neglected, especially under the intermediate water depth condition. The most frequently used numerical method for wave scattering and diffraction is the finite difference method (FDM) and finite element method (FEM), which can be classified as the domain method. Unfortunately, the FDM often cannot fit complex geometry boundary very well. On the contrary, the major advantage of the FEM is that it can handle complicated geometries without difficulty. However, for the computation