ABSTRACT

This article applied a general laminar flow model to simulate the cnoidal wave generation and its induced oscillatory flow over a wavy bed. The numerical model based on the stream function-vorticity \((\psi - \omega)\) formulation by the finite analytic (FA) discretization method is used. The time-evolving boundary-fitted grid system is adopted in order to conform with the transient free surface and the moving wave plate all the time, where the fully nonlinear conditions are treated. In this study, the cnoidal waves developed to its final permanent waveform on a flat bottom were generated by imposing the wave plate in motion. We thus investigated those unsteady near-bottom flow characteristics influenced by the roughness of wavy bottom. Numerical results are compared with analytical solutions presented by Tanaka et al. (1998) and experimental data conducted by Fredsøe et al. (1999). From the viscous flow near the wavy wall due to vortex shedding and flow separation, the interacting flow pattern, the oscillatory boundary-layer characteristics, and flow particle trajectories were investigated in detail herein.

KEY WORDS: cnoidal waves; wavy bed (ripples); stream function-vorticity; transient body-fitted grid; finite analytic method

INTRODUCTION

Flat sea bed is rare in nature. The hydraulic roughness, or equivalent Nikuradse roughness, exhibits its scale to be the order of 100 to 200 grain diameters for a flat mobile sand bed in an oscillatory sheet flow from the available friction and energy dissipation data (Nielsen, 1992). This is surprisingly different from the corresponding roughness, which is generally one order of magnitude smaller, to a steady sheet flow. As covered by sedimentary material of a wide range of grain sizes, the sea bed possesses complicated geometrical structures of many different shapes. These structures, including for example, bars, dunes, ripples, etc., keep interacting with the flow field in several different ways. Wavy beds (or sea bed ripples) are present on sand beds in shallow water waves, and are formed by the to-and-fro flow induced by wave motion above the bed. Although they do not have a major impact on the primary wave pattern if the ripples are not high, wavy beds can strongly influence the flow field of the benthic boundary layer. Therefore, they play a significant role on the transport processes in vortex generation and convection, sediment activation and movement, shear-flow friction, wave dissipation and pollutant dispersion, etc. Numerous researches are in connection with sediment transport in the shallow coastal region and many of these studies are, however, restricted to their application based on linear waves or purely harmonic oscillations. These results can not directly applied to sediment movement and describe the flow pattern well in near-bottom region, because the asymmetry of the wave profile is an important factor to them (Tanaka et al., 1998). Cnoidal wave theory contains non-ignorable effects of nonlinearity in addition to those of wave dispersion already included in linear wave theory. Even for the first-order cnoidal wave theory it can describe such nonlinear effects to some extent, whereas the linear wave theory cannot (Isobe, 1985). In this respect, the cnoidal wave theory might be more suitable to describe near bottom characteristics and sediment transport in nearshore current. However, due to mathematic complexity related to the cnoidal wave theory, there are relatively few studies on boundary layer flow under the motion of cnoidal waves.

Due to their engineering significance, wavy beds formed by wave motion have been widely studied both experimentally and theoretically. Ayrton (1910) might be the first to investigate the ripple formation. He recognized that the formation of sand ripples is connected with the vortices around the crests of the ripples, in alternate directions for every half-cycle. Bagnold (1946) demonstrated that the existence of ripples is a consequence of water motion above the sandy bottom. When the relative flow speed exceeded a certain threshold, most of the sand particle on top of the sand layer began to roll, and soon to form ripple crests aligned normal to the direction of oscillation. These phenomena are called rolling-grain ripples. If the speed of the oscillatory water over the ripples already formed is increased sufficiently, vortices start to form in the water behind the ripple crests, dislodging particles into suspension and further sharpening the ripple profile. Bagnold called these the vortex ripples. Some experiments have been conducted in wave flumes or water tunnels to measure some relevant features of flow like the mass transport velocity, the vorticity and the amount of sediment transport, etc (Carstens and Neilson (1967), Fredsøe et al. (1999), Marin (2004), etc.). On theoretical works, analytical studies on the transport immediately above a ripple bed were relatively few and mostly limited to two-dimensional unidirectional flow (Liu (1957), Kennedy (1963), Engelund (1970), Fredsøe (1974), Richards (1980), Sumer and Bakioglu (1984), etc.). Although oscillatory flow above a ripple bed has been studied for many years, there is still no universally unified theory to fit all cases. Due to theoretical and experimental difficulties, numerical models become widely adopted after 1990’s. Two-dimensional models were developed,