Nonlinear Surface Waves in a Nearly Square Liquid Tank under Vertical Excitation

Takashi Ikeda
Mechanical System Engineering, Graduate School of Engineering, Hiroshima University
Higashi-Hiroshima, Hiroshima, Japan

Raouf A. Ibrahim
Department of Mechanical Engineering, Wayne State University
Detroit, Michigan, U.S.A.

ABSTRACT

This paper investigates the parametric excitation of a liquid surface in a nearly square tank when the 1:1 internal resonance is nearly satisfied. The two sloshing modes, \((1,0)\) and \((0,1)\) are nonlinearly coupled. The dependence of amplitude-frequency response curves on the liquid level, the excitation amplitude, and the aspect ratio of the tank length and breadth are analytically and numerically predicted. The predicted results are compared with those measured experimentally and some discrepancies were observed.

KEY WORDS: Liquid sloshing; nearly square tank; vertical excitation; internal resonance, frequency response curve

NOMENCLATURE

- \(a\): excitation amplitude
- \(d\): tank breadth
- \(g\): acceleration of gravity
- \(h\): liquid level
- \(l\): tank length
- \(P\): fluid pressure
- \(p_{mn}\): natural frequency of sloshing mode \((m, n)\)
- \(t\): time
- \((x, y, z)\): Cartesian coordinate
- \(z_0\): vertical translation of the tank
- \(\zeta_{mn}\): damping ratio of sloshing mode \((m, n)\)
- \(\eta\): liquid elevation
- \(\rho\): fluid density
- \(\phi\): velocity potential
- \(\omega\): excitation frequency

INTRODUCTION

The problem of liquid sloshing in moving or stationary containers remains of great concern to aerospace, civil, and nuclear engineers; physicists, designers of road tankers and ship tankers; and mathematicians. Civil engineers and seismologists have been studying liquid sloshing effects on large dams, oil tanks and elevated water towers under ground motion. Liquid tanks are also mounted on the roofs of multi-story buildings as a means of controlling building vibrations caused by earthquakes. One of the most recent and serious problems of liquid sloshing dynamics occurred during the July 2007 earthquake in Japan. This earthquake caused a small amount of contaminated water to spill over the side of a spent fuel storage pool in one of the nuclear power plants and caused severe damage. The spectrum of any earthquake contains a wide range of frequencies, one of which can induce large liquid amplitude oscillations if it is near one of the natural frequencies of sloshing. Thus, it is imperative to study the behavior of liquid surfaces under sinusoidal excitation. In addition, one must consider the nonlinearity of the liquid dynamic behavior in the theoretical analysis.

Sloshing waves can be generated when the liquid container is vertically excited at a frequency close to twice the natural frequency of the free surface, as observed by Faraday (1831). Faraday waves occur at one half of the excitation frequency. Mathiessen (1868, 1870) conducted a series of experimental investigations and showed that the fluid oscillations are synchronous with the excitation frequency. The contradiction of the two observations led Lord Rayleigh (1883a, b, 1887) to make a further series of experiments with improved equipment and his observations supported Faraday’s results. During that time, Mathieu (1868) formulated his equations, which helped Rayleigh to explain this phenomenon mathematically. The contradiction was investigated again by Lewis (1950), Taylor (1950), Benjamin and Ursell (1954), Konstantinov, et al. (1978), and Nevolin (1985) who also explained mathematically the discrepancy between Faraday’s and Mathiessen’s observations. Dodge, et al. (1965) observed harmonic and double superharmonic liquid motions. The historical developments of Faraday waves including multi-mode interactions in circular, rectangular and nearly square containers are well documented in Ibrahim (2005).

Miles (1984a, b) studied sloshing modal interaction in circular containers when two modal frequencies are equal. Miles (1984a), Holmes (1986), Gu and Sethna (1987), and Gu, et al. (1988) studied the case of parametric excitation when the modal frequencies are in the ratio of 1:2. Holmes (1986) qualitatively showed the existence of chaotic motions for certain parameter ranges close to the 2:1 subharmonic resonances. Gu and Sethna (1987) studied periodic, almost periodic, and chaotic wave motions in a rectangular tank subjected to vertical sinusoidal excitation. The internal resonance