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Numerical Simulations of Shallow-water Wave Propagation over Arbitrary Bottom Topography using High-order Boussinesq Equation

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ABSTRACT

The propagation of shallow water waves is simulated using high-order Boussinesq equation, which approximates the flow velocity by series expansions. In this study, the expansion is truncated at the 3rd order (Boussinesq-type) involving up to the fifth derivative operators. The finite series are constructed with the help of the Padé approximation in order to ensure the accuracy of the highest degree for a given number of terms. Through this, the accuracy of the dispersion property in the wave propagation is assured up to the wave numbers of as high as $kh = 25$, and at the same time, the effectiveness of the vertical structure of the velocity field is enhanced for kh of up to 12. The numerical scheme is implemented based on the time stepping integration of the exact surface boundary conditions. Numerical models are constructed in two dimensions (thus three dimensions for velocity) in order to closely examine the flow characteristics such as the generation, propagation and absorption of shallow water waves. The numerical results are found to coincide quite well with the exact solution and also with experimental data. The wave refraction and shoaling due to bottom topography in coastal regions are also investigated.

KEY WORDS: Shallow water wave; Boussinesq equations; Padé approximation; Finite difference method; shoaling; refraction.

INTRODUCTION

Wave effects are important considerations in the analysis of coastal structures such as breakwaters, harbor, and moored-floaters. The design of these structures generally considers a number of possible wave conditions in order to determine the design criteria. Wave conditions can be generated by suitable mathematical models or numerical methods which must cope with various wave deformations such as shoaling, refraction, diffraction and reflection of waves propagating from deep water to shallow water.

Boussinesq models are well known to be the most accurate method for describing the propagation of non-linear shallow water waves near coastal regions. The Boussinesq formulation is well known for its incorporation of dynamic properties into horizontal dimensions by eliminating the vertical coordinate. It significantly reduces the computational burden relative to three-dimensional methods, enabling wave simulations in a wide coastal region to be implemented. Boussinesq (1872) equation was derived by eliminating the vertical dependency and assuming $O(\mu^2) = O(\varepsilon) < 1$, where $\mu = k_0 h_0$, $\varepsilon = a_0 / h_0$. k_o, a_0 and h_0 are the typical wave number, amplitude and water depth in this order at a far upstream reference location. Based on perturbation theory, Mei and LeMéhauté (1966) and Peregine (1967) modified the Boussinesq equation valid for waters of variable depth. Although these models contained the dispersive term, its range of validity is limited to very shallow water because its linear dispersion relationship is only a rough approximation of the exact one.

In order to apply the Boussinesq equation to waves propagating in intermediate or deep water, the modified Boussinesq equations with improved dispersion characteristics are suggested. Witting (1984) obtained the rational polynomial expansion of the linear dispersion relation. Madsen et al (1992) included higher-order terms with adjustable coefficients into the standard Boussinesq equation for even and variable bottoms. The accuracy of the dispersion relation could be considerably enhanced by introducing the Padé(2,2) approximant with appropriate coefficients. Gobbi et al. (2000) derived a Boussinesq model accurate to $O(\mu^4)$ in dispersion and retaining all nonlinear terms. They used the weighted average of the velocity potential at two distinct water depths to obtain highly accurate dispersion to the Padé(4,4) approximant. Agnon et al. (1999) presented an exact formulation of the boundary conditions at the free surface and the bottom combined with an approximate solution to the Laplace equation given in terms of truncated series expansions. The resulting velocity fields are expressed in terms of velocity components of still-water data. As a result, this formulation makes it possible to obtain an accurate description of dispersive waves with non-linear terms up to $kh = 6$.

The methods mentioned above, however, do not provide an accurate vertical distribution of the velocity field. Madsen et al. (2002) suggested a new type of non-linear wave equations retaining the vertical velocity as an unknown. In this method, the Laplace solution is extended from an arbitrary z-level rather than the still-water data, which is quite different from the conventional Boussinesq equations. His fifth-order model can describe highly non-linear waves to $kh = 25$ for dispersion property, with accurate velocity profiles up to $kh = 12$. It means that waves from deep to shallow water can be simulated with this method, but for the initial step only shallow and intermediate water