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Analysis of Elastostatic Crack Problems using B-spline Wavelet Finite Element Method

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ABSTRACT

B-spline wavelet finite element method to the elastostatic crack problems is proposed. B-spline wavelet finite element method (Tanaka et. al., 2006) is a new technique for solving solid/structural mechanics problems. B-spline scaling function and wavelets are used as the basis functions in the Galerkin formulation. These wavelet basis functions have the so-called multiresolution properties. The solutions can be refined in regions of high gradients such as stresses or strains by superposing different lengths scale wavelets. Adaptive strategies using the multiresolution properties are developed for the elastostatic solid/structural problems (Tanaka et. al., 2007). On the other hand, there are difficulties in dealing with discontinuous displacements for the analysis of crack problems because the continuities of the displacements are always guaranteed the B-spline wavelet finite element method. Then, we propose a new technique. The displacements are enriched by discontinuous functions and the near tip asymptotic solutions through a partition of unity method (Melenk, 1996; Babuska, 1997). In this paper, emphasis is placed on linear B-spline scaling function as the basis functions in Galerkin formulation. Mathematical formulation and numerical implementations for the crack problems are presented. Some numerical examples for the two dimensional elastostatic crack problems are shown.

KEY WORDS: Wavelet, Finite Element Method, Fracture Mechanics Analysis, Stress Intensity Factor.

INTRODUCTION

The objective of this study is to solve the solid/structural problems using B-spline wavelet finite element method. It has been known that the use of wavelet is a new technique for solving partial differential equations, as well as signal processing, image processing and function approximation (Charles, 1992; Williams, 1994). A signal or a function is interpolated by scaling function and wavelets in the wavelet analysis. Wavelet basis functions can divide a given signal or function into different length scale component. These features of the wavelet basis functions are the so-called multiresolution properties. A various kinds scaling function and wavelets have been proposed for different purposes. Among them, B-spline scaling function and wavelets (Charles, 1991) are used in present study because they are very tractable compared with other wavelet basis functions.

To discretize the elastostatic body by the B-spline wavelet finite element method, B-spline scaling function and wavelets are placed periodically along the coordinate directions. The body is divided into equally spaced structured cells for the integration of the stiffness matrices. The interior and exterior of the body are represented by performing and not performing the integration. There are no mesh generation processes in this method. It is relatively easy to generate an analysis model for a complex shaped structure. Then, B-spline wavelet finite element method can be considered to be a kind of voxcel finite element method (Hollister et. al., 1994). Furthermore, the multiresolution properties of the basis functions can enhance the efficiency and accuracy. High gradients of stresses or strains can be enhanced by superposing finer length scale wavelets locally. Adaptive analysis using the mulitiresolution properties are developed for the elastostatic solid/structural analyses (Tanaka et. al., 2007). On the other hand, there are difficulties in dealing with discontinuous displacements such as in crack problems. In order to overcome these difficulties, a new technique to treat discontinuous displacements is proposed. The displacements are enriched by the discontinuous function and near tip asymptotic functions (Fleming, 1997; Belytschko, 1999) through a partition of unity method. This technique does not require cells to conform to the crack geometry. The stress concentration region around the crack tip can be represented by superposing finer length scale wavelets and the enrichment functions. It is expected that simulation of crack propagation in complex shaped structure is able to be carried out without remeshing. In this paper, as our first step, emphasis is placed on the linear B-spline scaing function as the basis functions. The accuracy of B-spline wavelet finite element analyses for the crack problems is evaluated.

The outline of the paper is as follows: First, the mathematical formulation and the numerical implementation of the B-spline wavelet finite element method for the elastostatic crack problems are shown. Stress intensity factors for mixed-mode crack problems are evaluated