Proceedings of the Eighteenth (2008) International Offshore and Polar Engineering Conference
Vancouver, BC, Canada, July 6-11, 2008

ISBN 978-1-880653-70-8 (Set); ISBN 1-880653-68-0 (Set)

A Numerical Study of Probability Distribution of Nonlinear Responses

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ABSTRACT

The responses of offshore structures in random seas are stochastic processes. The extreme values and cycle count of the responses, used to assess the strength of the structures, are mostly calculated based on the assumption that the responses fit a Gaussian distribution. If nonlinear waves or nonlinear motions are involved, the assumption of the Gaussian distribution may not hold. This paper will present an investigation into how well the Gaussian assumption works for the response of a nonlinear system excited by a Gaussian white noise. A numerical method is proposed to solve the probability distribution of the nonlinear response from the Fokker-Planck equation. The cycle count of the response can be calculated from the numerical solution. By comparison with simulated data, it shows the formula based on the Gaussian assumption often overestimates cycle count and the proposed numerical method can give an accurate prediction of probability distribution and cycle count for the nonlinear response.

KEY WORDS: Nonlinear response; stochastic processes; probability distribution; cycle count; finite difference method.

INTRODUCTION

After experiencing several heavy storms such as Katrina and Rita in the Gulf of Mexico, people in offshore industries now pay more attention to nonlinear hydrodynamics. With today's fast computers, it is possible to model nonlinear waves and solve nonlinear motions of offshore structures by direct simulations. The rest of the work is to find some criteria such as stress cycle count and extreme values of stress from the simulations to assess the strength of the offshore structures. As the responses of a system in random seas are stochastic processes, these criteria are commonly calculated based on the assumption that the responses are Gaussian processes, although this assumption may not hold if nonlinear waves or nonlinear motions are involved. The purpose of this paper is to study the errors due to the Gaussian assumption and to find an alternative method to predict probability distribution for a particular nonlinear system.

In general, there is no analytical solution available for the probability density function (PDF) of a nonlinear response. Nevertheless, if the driving force of a system can be treated as a Gaussian white noise, then the response of this system can be modeled as a Markov process and its transitional PDF can be found in the form of a partial differential equation called the Fokker-Planck equation. Except for a few simple cases, the Fokker-Planck equation has to be solved numerically. Haddara and Zhang (1991) have solved this equation using a Garlerkin method with Hermite polynomials to obtain the PDF of nonlinear roll motions of a ship. The limitation of this method is that the terms in the motion equation have to be polynomials. In recent years, many numerical methods have been developed to solve the Fokker-Planck equation, such as finite element method (Bergman, et al., 1996)), finite difference method (Johnson et al., 1997) and path integration method (Naess, et al., 2007). Since most of the time the low probability region is of more interest, such as in the case of an extreme value problem, the accuracy of a numerical method is critical. In addition, the efficiency of the numerical method is also a major consideration, especially for problems of high dimensional joint probability distribution.

In this paper, a second order nonlinear dynamical system with a Gaussian white noise excitation is studied. A difference method with a more efficient and accurate scheme is presented to solve the Fokker-Planck equation. Verified by benchmark solution and simulated data, it shows that the proposed numerical method can accurately predict probability distribution and cycle count of the response for this nonlinear system. From numerical simulations, the nonlinear effects to the probability distribution and cycle count are demonstrated by varying nonlinear contributions in the system. The comparison of cycle count calculated by Gaussian assumption with that from simulated data shows the Gaussian assumption tends to overestimate the cycle count.

GOVERNING EQUATIONS AND NUMERICAL SOLUTION

A general second order dynamical system driven by a Gaussian white noise can be written as: