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# Theoretical Modelling Of The Kinematics of Extreme Random-Wave Event generated by Focusing.

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## ABSTRACT

Prediction of particle kinematics in extreme wave events generated by focusing is still difficult. The semi-theoretical method of Grue et al (2004) over-predicts velocities below MWL in extreme cases and a purely theoretical method is still lacking. Thus a theoretical method is presented based on higher order terms and a simplistic wave-wave interaction. Results of the new method are compared with those of Baldock et al(1996) and by extension, those of Grue et al(2004) whose results corroborate those of Baldock et al(1996). The comparison shows a surprisingly good agreement with measurements and very significant improvement over existing methods.

## KEY WORDS:

Random-waves; Interaction; Kinematics; Extreme-waves; Non-linear.

## NOMENCLATURE

MWL-mean water level
ak – steepness (local crest elevation multiplied by local wave-number)

## 1.0 INTRODUCTION

The study of extreme waves has become very important with the recording of the "new-year" wave at the Draupner platform in the north-sea. Investigation of these extreme kinds of waves has depended on extreme events generated by focusing wave components of an irregular sea. Unfortunately, the occurrence of such a phenomenon is highly stochastic and cannot be deterministically obtained.

Tromans et al (1991) presented the new-wave theory which predicts the most probable extreme wave elevation given the spectral properties of the sea-state. However, the method is only valid to first order and does not take into account the non-linearities inherent in very steep waves as seen in the extreme events. Indeed, Baldock et al(1996) have shown that the focusing of wave components produce a highly non-linear wave group in which water surface elevation and near-surface particle kinematics are significantly larger than those predicted by the linear sum of the wave components, as done in the classical linear theory. As a result, Walker et al (2005) have extended the new-wave theory to the $5^{th}$ order and have used it to predict the elevation of the "new-year" wave with reasonable accuracy. While the prediction of focusing wave surface elevation has become easier with the work of Walker et al (2005), prediction of the underlying kinematics is still very difficult. Only Grue et al (2004) have presented a semi-theoretical method which uses measured crest-heights and wave-periods to obtain kinematics. However, a purely theoretical method is still lacking and even the method predicted by Grue et al (2004) although giving very good results above MWL, over-predicts particle velocities below MWL in the steeper waves. In this work we develop a purely theoretical expression for wave particle velocity using the same approach Walker et al (2005) have used to predict water surface elevation. The approach extends linear new-wave theory to the fifth-order by combining the linear description of the elevation of the wave and its Hilbert transform using Stokes-type corrections to approximately but robustly account for the higher order contributions.

## 2.0 HIGHER-ORDER STOKES WAVES

Fenton (1985) considered regular waves (propagated without change of form over a layer of fluid) on a horizontal impermeable bed, where the origin is on the bed, the horizontal coordinate is x and the vertical coordinate is y, on a moving reference frame. By assuming incompressibility he introduced a stream function $\psi$ such that velocity components u and v in the x and y directions respectively, could be written as $u = \delta\psi/\ \delta x$ and $v = -\delta\psi/\ \delta y$ . Fenton further assumed irrotational flow so that $\psi$ satisfied the Laplace equation i.e

$$\delta^2\psi/\ \delta x^2 + \delta^2\psi/\ \delta y^2 = 0$$

On the free-surface $y = \zeta(x)$, the kinematic boundary condition applies:

$\Psi[x,\ \zeta(x)] = -Q$ where Q is a position constant denoting the total volume flow-rate underneath the stationary wave per unit length normal to the x-y plane.
By requiring constant pressure on the free-surface and combining with Bernoulli's equation, we have:

$$½[\ \delta^2\psi/\ \delta x^2 + \delta^2\psi/\ \delta y^2\ ] + g\zeta = R$$