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Finite Volume – Finite Difference Scheme for the Solution of 2D Extended Boussinesq Equations

Mara Tonelli *, *Marco Petti*
Dipartimento di Georisorse e Territorio, Università degli Studi di Udine.
Udine, Italia.

ABSTRACT

The paper presents a hybrid scheme for the solution of 2DH extended Boussinesq equations. The finite volume method is applied to the advective part of the equations, while dispersive and source terms are discretized by the finite difference technique. To validate the numerical model, a classical refraction-diffraction test is proposed. Special attention is devoted to verify the shock-capturing capabilities of the scheme: the model is applied to one- and two- dimensional runup test cases with good results, showing that no ad hoc treatment is required at the shoreline.

KEY WORDS: Boussinesq Equations; hybrid scheme; shock-capturing methods.

INTRODUCTION

In the latest decades two different approaches have been developed in the study of nearshore hydrodynamics: one is based on the solution of Boussinesq-type equations, the other on the use of De Saint Venant or nonlinear shallow water equations (NSWE), a non-dispersive subset of the former.

Boussinesq-type equations have proved to be a powerful and well-tested means for the simulation of wave propagation in coastal areas. The standard Boussinesq equations for variable bathymetry were first derived by Peregrine (1967), retaining the lowest-order effects of nonlinearity and dispersion. Owing to the poor dispersive properties, standard equations could only be applied to shallow water areas, a severe restriction for most practical problems. Considerable effort has been made to improve the dispersive characteristics of standard equations, rearranging the dispersive terms or using different velocity variables (eg. Madsen et al., 1991; Nwogu, 1993). These methods of derivation lead to sets of equations, referred to as extended or improved, whose dispersion relations are closer to the exact linear dispersion relation in intermediate water than standard equations. Recently new higher-order formulations have been derived, extending the range of applicability of Boussinesq-type equations to highly nonlinear and dispersive waves (Madsen et al., 2006) but the integration of these sets of equations is still computationally expensive.

NSWE are not suitable to simulate wave propagation because the waves get deformed over long distances, due to the absence of dispersion; their application is therefore restricted to very long waves. Nevertheless NSWE have been widely used in the inner surf zone to model nearshore dynamics like low-frequency waves generation (Watson and Peregrine, 1992) and shoreline motion (Brocchini et al., 2001). The main advantage of NSWE lies in their suitability to be integrated by means of shock capturing techniques, like the finite volume method (FVM), which allow an intrinsic representation of flow discontinuities.

Most Boussinesq models are based on the finite difference (FDM) or the finite element methods, so the numerical treatment of discontinuous phenomena requires the introduction of artificial techniques. To account for the effects of wave breaking, additional terms computed by the surface roller method (Madsen et al., 1997) or the eddy viscosity method (Kennedy et al. 2000) are introduced in the equations. In both cases the estimation of empirical parameters is necessary, thus a certain arbitrariness cannot be avoided. The simulation of shoreline motion has been performed coupling the numerical models to different moving boundary algorithms. Madsen et al. (1997) and Kennedy et al. (2000) applied the slot or permeable-bed technique; some authors (eg. Zelt, 1991) used a Lagrangian approach to follow the shoreline boundary movement and, in the past years, extrapolating boundary techniques have been implemented into Boussinesq-type models by Lynett et al. (2002) and Fuhrman and Madsen (2008).

The application of FVM to Boussinesq-type equations is not straightforward, due to the presence of dispersive terms, which have to be treated at a high-order accuracy level. Recently, a new approach based on a hybrid scheme, combining finite volume and finite difference methods, has been utilized to integrate 1D extended Boussinesq equations (Erduran et al., 2005). Extending the model of Erduran to the surf zone, Petti and Bosa (2007) proved that the shock-capturing abilities of finite volumes allow the automatic representation of spilling breakers.

The presented scheme associates FVM and FDM to solve the enhanced 2D Boussinesq equations proposed by Madsen and Sørensen (1992); the finite volume method is applied to the convective part of the equations, while dispersive and source terms are discretized by finite differencing. Water depth is calculated explicitly from the continuity equation; momentum equations are rewritten introducing a transformation of variables, thus fluxes are determined implicitly. Time