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Computation of reflection coefficient using wave amplitude and phase function

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ABSTRACT

The mild-slope equation and modified mild-slope equation are solved in a progressive manner, the boundary conditions being provided at a side. We introduce the complex triangular relationship between the velocity potential function and the wave amplitude and the phase gradient function. Then the modified mild-slope equation is transformed into a set of equations by separating real and imaginary numbers. Incident waves propagate in the *x* direction, while reflected waves propagate in the negative *x* direction. Transmitted waves propagate in the x direction, too. We provide boundary conditions at the right end of the computation domain, where the wave amplitude, gradient of the wave amplitude, and the phase gradient are given. A spatially and temporarily central numerical scheme is used to solve the two equations. Solutions are obtained from right end to the left end. Once the distributions of the wave amplitude and the phase gradient are obtained, we compute the reflection coefficient from the maximum wave amplitude and the minimum wave amplitude around the left boundary where the incident waves and the reflected waves superimpose. Reflection coefficients are computed for inclined plane beds, smoothly changing depth beds with a hyperbolic tangential function, and sinusoidally wavy beds by solving the modified mild-slope equation. The spatial distribution of the computed wave amplitude is sinusoidal at the wave-incoming zone before bathymetric variation. The computed spatial distribution of the phase gradient is also sinusoidal at the wave-incoming zone. The computed relationship between the step width and the reflection coefficient is compared with the results for the original mild-slope equation. The computed results demonstrate that the present computation confirms previous findings, one of which is that the gap between reflection coefficients of the two equations is large for a small step width.

KEY WORDS: mild-slope equation; modified mild-slope equation; wave amplitude; wave phase function; reflection coefficient; centered scheme.

INTRODUCTION

The mild-slope equation has been widely used for the description of wave transformation over mild-sloped sea beds since it was proposed by Berkhoff (1973). A sea bed is often called mild-sloped when the bed slope is much smaller than 1. The mild-slope equation was developed from either the continuity equation or the principle of stationary action by including the variational principle. The accuracy of the mild-slope equation is known to be guaranteed up to a certain bed slope. The mild-slope equation has also been proposed in different types of partial differential equation by Radder (1979) and Copeland (1985).

More recently the modified mild-slope equation was proposed by Massel (1993), and Chamberlain and Porter (1995). Two time-dependent forms of the modified mild-slope equation were presented by Suh et al. (1997) by using Green's theorem, and the variational principle which was used by Luke (1966) to confirm the Eulerian equations of motion for the classical water wave problem. Suh et al.'s equations are transformed into the modified mild-slope equation of Massel when the time-dependent term is replaced by a time-invariant term. Suh et al. suggested that the modified mild-slope equation could also be used for random waves by involving the carrier frequency. The modified mild-slope equation is reduced to the mild-slope equation when some higher-order terms of the modified mild-slope equation are turned off.

The modified mild-slope equation is known to have the applicability for a wider range of bed slopes than the mild-slope equation. Furthermore the modified mild-slope equation has produced more accurate reflection coefficients for Bragg's sinusoidal bed tests than the mild-slope equation due to the higher-order terms in it.

For one-dimensional problem the time-invariant forms of the mild-slope and modified mild-slope equations become second-order ordinary differential equations, and have been solved with boundary conditions at both side boundaries. It is demonstrated here that the mild-slope and modified mild-slope equations can be solved in a progressive manner in case adequate boundary conditions are provided at one side only.

MILD-SLOPE AND MODIFIED MILD-SLOPE EQUATIONS

Before transforming the mild-slope equation into another form we briefly summarize Berkhoff's derivation of the mild-slope equation here. Berkhoff (1973) derived the mild-slope equation starting from the continuity equation. Irrotational free surface water waves of incompressible fluid water were considered. A velocity potential