

A Viscous Free Surface Flow Solver Using the Spectral Element Method

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ABSTRACT

A viscous free surface flow solver is introduced in this paper. The present model of the fluid motion is based on the Navier-Stokes equations of velocity-pressure formulation. In order to deal with the free surface motion, an Arbitrary Lagrangian-Eulerian (ALE) description is adopted. The governing equations are solved through the Spectral Element Method (SEM), which possesses the property of high-order spatial accuracy as proposed by Karniadakis & Sherwin (2005). The solution procedure and characteristic aspects of the present modeling and numerical method are briefly addressed. The essential part of the present method is focused upon the treatment of kinematic and dynamic free surface boundary conditions.

It is shown that the proposed SEM and its present implementation produce very accurate and convergent solutions to several test problems including manufactured one of N-S equations. Numerical results of the lid-driven cavity flow are also compared with other numerical results. As for the viscous free surface flows, viscous sloshing with a prescribed initial free surface profile and run-up of soliton on vertical wall are simulated to check the accuracy of the present method. Generation of nonlinear free-surface waves by applying pressure distribution on some portion of the free surface is realized in our numerical wave flume.

KEY WORDS: Viscous free surface; Navier-Stokes equations; Arbitrary Lagrangian-Eulerian (ALE); Spectral Element Method (SEM); viscous sloshing; soliton; nonlinear free surface waves.

INTRODUCTION

Most analyses of the free surface flows in the field of naval hydrodynamics and ocean wave mechanics have been utilizing the potential fluid flow model with and without body presence as manifested in the research papers of Grilli et al. (1989), Scorpio et al. (1997), Ferrant (1997), Celebi et al. (1998), Sung et al. (1998), Liu et al. (2001), Xue et al. (2001), and Sung & Grilli (2005). We can also find that there were a few attempts to deal with the free surface phenomenon by considering the flow irrotationality as well as fluid viscosity, which was sincerely tried in Yeung and Ananthakrishnan (1984), Armenio (1997), Hodges and Street (1999), Zhu et al. (2001), and Park et al. (2003), etc.

On the other hand in recent years, it seems that there is still continuing interest in viscous free surface flows in the field of computational fluid dynamics. As denoted in Robertson et al. (2004), viscous effects are needed to be included for the flows such as (1) a

flow past a body at high Keulegan-Carpenter number where the free surface influences locally, (2) wave generation of submerged and floating bodies, and (3) damping of sloshing waves in a container. As far as we are concerned with viscous effects, our primary concern lies on the free surface flows to which we are not able to apply the classical potential theory.

Currently, as indicated in Sung et al. (2006 and 2007), there has been necessity of developing an accurate numerical method for wave-current interaction problem, which was motivated by the freak wave generation mechanism. As for the case of uniform currents, the potential flow theory is sufficient even in the nonlinear regime. On the other hand, being varying spatially, non-uniform currents which we suppose to be one gateway to freak waves do not allow the classical potential flow theory (Thomas and Klopman, 1997). Thus we need to have an appropriate fluid model of wave interaction with non-uniform current for better understanding and prediction. Because the combined flow field will be rotational, we should use the Euler equation model or the Navier-Stokes (N-S) equations. Though we can utilize the stream function approach, it is limited to the two dimensions. Therefore we assume that the Euler or N-S equations are to be preferred for generality and versatility. Another reason is that many reliable and robust theories with efficient numerical methods have been developed for the Euler and N-S equations, and so we can resort to them with slight modification and extension for free surface problems. Since viscosity effects need to be included for the bottom friction and atmospheric forcing, etc., the flow model of the present study is based on the viscous incompressible flow. Hence the governing equations are the Navier-Stokes equations.

The present paper consists as follows. Flow model and numerical method are introduced. Solution procedure and characteristic aspects of the present modeling and numerical method are briefly addressed. Accuracy and rate of convergence of the proposed SEM and its present implementation are demonstrated for several test problems. Finally numerical results of a few selected problems of viscous free surface flows show that the present methodology works quite well and that it is efficient and accurate.

FLOW MODEL AND GOVERNING EQUATION

As previously stated, the present flow problem is the unsteady viscous free surface motion under gravity in a bounded fluid domain, which is depicted in Fig. 1. Surface waves can be generated by wave-maker motion at a side wall of the domain or applied pressure distribution on some part of the free surface, while a mean flow or current can be included by imposing the inflow and outflow conditions.