

## General Solution of Spatial Warping Curved Beams under Multiple Loads

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### ABSTRACT

The purpose of this paper is to provide a general solution of spatial warping curved beams under multiple loads based on the existed theory. The transverse shear deformation and torsion-related warping effects are taken into account. In natural coordinates, equilibrium, geometrical and constitutive equations are established in matrix form. The analytical expressions to the problems can be obtained by solving the matrix equations. The internal forces, stresses, strains, and displacements are calculated along the beam axis. By using these solutions, a plane curved beam subjected to uniform vertical loads and torsions is analyzed. The accuracy and the efficiency of the present theory are demonstrated by comparing between the results with the solution of C.P.Heins.

**KEY WORDS:** Curved beam; multiple loads; warping; natural coordinate; analytical solution.

### INTRODUCTION

Curved beams have been applied in many engineering structures such as bridges, highway constructions, long span roof structures and machinery structures, etc. Therefore, many scientists have tried to analyze and solve problems of curved beams by different methods. The majority work in theoretical method aimed to the plane curved beams where deformations include out-of-plane bending and torsion. Analytical solutions of internal forces and deformations were derived (Heins, 1981; Yao, 1989) for plane curved beams with constant curvature under out-of-plane loads, based on traditional mechanics of structures, in which the coupling of flexure and torsion was taken into account. Further, Tufekci and Dogruer (2006) obtained the exact solution of out-of-plane problems of a plane arch with varying curvature and cross section. In order to investigate the reason of in-plane damage happened in curved bridges in recent years, Li, Liu and Zhao (2007) and Li and Zhao (in press) presented the exact solution of the displacement of the plane curved beams subjected to an in-plane load and a changed temperature. Along with the development of the study, some theoretical results for plane curved beams have been applied in the design of curved bridges (Shao and Xia, 1994; Gao Dao Chun Sheng, 1979; Sun, 1995). Recently the research interests focused

on the analytical solution of spatial curved beams. Xiong and Zhang (1997) established a set of governing equations for spatial curved rods under natural coordinates and provided the general solution for statically determinate supports. Zhu, Zhang and Yan (1999) discussed the bimoment of spatial curved bars of anisotropic materials with thin walled cross sections and obtained a general solving method for this type of problems. Yu and Yi (2004) presented a theoretical method for static analysis of naturally bended and twisted beams subjected to complicated loads by solving the equations in generalized coordinates, considering the effects of torsion-related warping as well as transverse shear deformations. Zheng, Zhang and Yan (2006) provided a matrix form solution for curved rods in natural coordinate system. Significant developments of theoretical analysis have taken place during the last decades, as outlined by Zhao, Kang, Feng and Lao (2006). However, because of the mathematical limitation, most of the studies above considering the effects of torsion-related warping could not provide explicit formulae. This brings really difficulties for application. The purpose of this study is to give the explicit solution of internal forces and deformations of the spatial curved beams under complicated loads, including the contributions of transverse shear deformation and torsion-related warping effect. The accuracy and efficiency of the solution are finally demonstrated by the analysis of an example.

### GOVERNING EQUATIONS AND SOLUTIONS

#### Exited Theories of Spatial Curved Beam

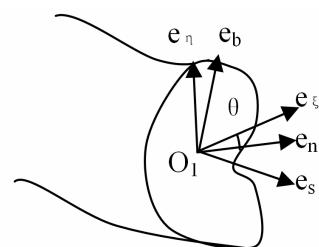


Fig.1. Spatial curved coordinates and geometry of the beam

Let the locus of the cross-sectional centroid of a spatial curved beam be a continuous spatial curve. The tangential, normal and bi\_normal unit