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The Lie-Group Shooting Method for Multiple-Solutions of Falkner-Skan Equation under Suction-Injection Conditions

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ABSTRACT: For the Falkner-Skan equation, including the Blasius equation as a special case, we will develop a new numerical technique, transforming the governing equation into a nonlinear second-order ordinary differential equation by a new transformation technique, and then solving it by the Lie-group shooting method. The second-order equation is singular, which is however much saving computational cost than the original equation defined in an infinite range. In order to overcome the singularity we consider a perturbed equation. The newly developed Lie-group shooting method allows us to search a missing initial slope at the left-end in a compact space of $t \in [0, 1]$, and moreover, the initial slope can be expressed as a closed-form function of $r \in (0, 1)$, where the best r is determined by matching the right-end boundry condition. All that makes the new method much superior than the conventional shooting method used in the boundary layer equation under boundary conditions imposed. When the initial slope is available we can apply the fourth order Runge-Kutta method to calculate the solution, which is highly accurate. The present method is very effective for searching multiple-solutions under very complex conditions of boundary suction or injection, and the motion of plate. The boundary layer flows studied here have broad applications in liquid film condensation, geophysical and oceangraphy contexts.

KEY WORDS: Falkner-Skan equation, Blasius equation, boundary layer, suction-injection boundary conditions, multiple-solutions, Lie-group shooting method.

INTRODUCTION

When an incompressible flow passes in the vicinity of solid boundaries, the Navier-Stokes equations can be reduced drastically. The boundary layer theory was first proposed by Prandtl in 1904. It asserts that the viscous effect would be confined to a thin shear layer adjacent to the solid boundary in the case of a motion with very little viscosity. Hence, the fluid motion is split into two parts: near the boundary the viscosity effect is important and the fluid is said to be viscous, and far away from the boundary the fluid viscous effect is unimportant, which can be treated with as an inviscous fluid.

The boundary layer theory explains very well the steady-state flow over a flat plate at zero incidence angle known as Blasius flow. A general transformation inspired by Meksyn (1961) permits us to transform the Prandtl equation into the Falkner-Skan equation. In this study we are focused on the Falkner-Skan equation:

$$f''' + ff'' + \beta(1 - f'^2) = 0 \tag{1}$$

with a suction or injection boundary condition and a moving solid boundary:

$$f(0) = -C, \quad f'(0) = \xi, \quad f'(\infty) = 1. \tag{2}$$

In the above, f denotes a normalized stream function with $df(\eta)/d\eta = f'(\eta) = u/u_1$ a normalized fluid velocity in the x-direction, and with u_1 the fluid velocity at the edge of the boundary layer. The independent variable η is a similarity variable, and $\xi = U_w/U_\infty$ is the velocity ratio. When $0 < \xi < 1$, the speed of oncoming fluid U_∞ is larger than U_w of the plate. When $\xi > 1$, the speed of the moving plate is faster than the speed of oncoming fluid, and $\xi = 0$ is for a resting plate. The term C is a constant related to suction if it is negative or injection if it is positive.

When $\beta = 0$, the Blasius equation is recovered, of which Callegari and Friedman (1968) have used a similar transformation and the Crocco transformation techniques to reduce the classical Blasius equation into a second-order nonlinear singular two-point boundary value problem (BVP), and assured the existence, uniqueness and analyticity of positive solution.

The solutions of Blasius and Falkner-Skan equations have been studied numerically by many researchers like as Cebeci and Keller (1971), Asaithambi (1998,2005), Elbarbary (2005),