

Transformations of Wave Passing around Cylinders

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ABSTRACT

This study investigates wave transformation when a wave passes over circular cylinders. The numerical model is based on the Boussinesq equation developed by Nwogu (1993), expressed by velocity with arbitrary water depth. The numerical model utilizes the Fourth-Order Adams-Bashforth-Moulton Predictor-Corrector Scheme and is combined with a source function and absorbing boundary condition to enhance calculations stability and reduce the required processing time. The absorbing boundary condition is a sponge layer combined with a radiation boundary condition. Several numerical experiments are made to simulate wave transformations over circular cylinders. The interactions among incident wave, reflected wave and scattered waves due to cylinder are obvious in wave height distributions.

KEY WORDS: Boussinesq equations; wave transformation; source function; absorbing boundary; circular cylinders; sponge layer; radiation boundary condition.

INTRODUCTION

Recently, Boussinesq equation has become the most popular equation in the prediction of wave transformations. Boussinesq (1872) derived the original Boussinesq equations. Thereafter, numerous researchers improved and extended their applicability. Peregrine (1967) considered various water depth conditions to derive a shallow-water wave equation from small amplitude wave theory. He used a depth-averaged velocity as the dependent variable to derive the Boussinesq equation at a constant water depth. Witting (1984) included the free surface velocity as the dependent variable in a nonlinear depth-integrated momentum equation. Expanding velocity terms as Taylor series yields a one-dimensional Boussinesq equation; however, the weakness of the equation is its limited applicability to constant-depth situations and the difficulty of applying it to two-dimensional cases.

Conventional Boussinesq equations are limited to relatively shallow water. Hence, many studies have focused on extending them to deeper water. McCowan (1985, 1987) calculated nonlinear wave propagations in shallow water using the Boussinesq equation. The error between the derived phase velocity and the linear phase velocity was under 5%, and the relative depth of the water was extended to 0.2. Madsen et al. (1991)

improved the Boussinesq equation to enable it to be applied to relatively deeper water. The improved Boussinesq equation has a better dispersive characteristic than the conventional Boussinesq equation. However, their derived equation could only be applied when the depth of the water was constant. Madsen and Sørensen (1992) derived a Boussinesq equation which was not restricted to constant depth. Their equations also simulated irregular waves passing over mild slope sea beds. Nwogu (1993) derived a Boussinesq equation with velocity at arbitrary depth as a dependent variable, and successfully extended the limits of applicability to a relative depth of 0.5. He derived the same dispersion relation as did in Madsen et al. (1991). Wei and Kirby (1995) numerically solved the equation derived by Nwogu (1993) using a fourth-order Adams-Bashforth-Moulton predictor-corrector scheme efficiently reducing the errors associated with the numerical calculations. Their model is the well-known WKGS model.

Schäffer and Madsen (1995) derived a Boussinesq equation that better described dispersive and shoaling characteristics of waves. They applied [4,4] Padé approximations to linearized Stokes waves to derive a Boussinesq equation. The relative water depth was thus extended to unity. Gobbi and Kirby (1999) included fifth-order terms in derivations to yield a one-dimension Boussinesq equation. They considered the dispersive characteristic of waves' traveling over the submerged breakwater. Also, their numerical results were very consistent with experimental findings. Gobbi et al. (2000) linearly combined two arbitrary water depth distributions to derive the Boussinesq equations. These equations are applicable up to $kh \approx 6$. Madsen et al. (2002) employed finite series solutions to expand the exact solutions of the Laplace equation in an arbitrary water depth, and the Padé approximation was applied when solving these equations. The parameter kh is applicable up to about 40. In the vertical variation of velocity, the limitation of kh was extended to about 12. These equations greatly approve the dispersion of the Boussinesq equations. The equations agree well with the experiments simulating the wave transformations before the critical wave breaking.

MATHEMATICAL FORMULATION

The governing equations are shown as below,