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High Harmonic Forces and Predicted Vibrations from Forced In-line and Cross-flow Cylinder Motions

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ABSTRACT

String-like ocean structures, such as deep water marine risers are susceptible to a condition of dual lock-in, where both the in-line and transverse natural frequencies are excited due to vortex shedding in the wake. This type of excitation can result in dominant, large amplitude third harmonic forces in the cross-flow direction that do not exist in conditions allowing only cross-flow motion. Forced motions of a rigid cylinder in both the in-line and cross-flow directions are performed to obtain coefficients defining the magnitude of third harmonic lift forces for given cylinder motions. In-line motion amplitude, cross-flow amplitude, phase between in-line and cross-flow motion, and reduced velocity are varied, producing a four-dimensional matrix of data points, at a Reynolds number of 8800. In free vibrations, variation of the effective added mass drives the system to specific steady-state oscillations, under lock-in conditions. These free vibration steady-state oscillations are successfully predicted with the new forced oscillation data set, using the simplifying assumption that lock-in occurs in both the in-line and cross-flow directions, and the necessary assumption that the normalized average power over one cycle must be zero for a free vibration. The new data set and our procedure will allow a more accurate strip-theory approach to marine riser VIV analysis and design.

KEY WORDS: VIV; vortex-induced vibration; 2 DOF; forced vibration

INTRODUCTION

As the demand for natural resources drives ocean structures, such as marine risers, to deep water, the necessity for an accurate understanding of the physics of the combined in-line and cross-flow oscillation of these structures due to vortex shedding becomes apparent. The fundamental problem of vortex-induced vibrations and a number of studies on the topic are discussed in various comprehensive reviews (Sarpkaya, 1979, 2004; Bearman, 1984; Williamson and Govardhan, 2004).

Ocean structures such as risers and cables typically have comparable structural mass compared with the displaced mass of fluid (mass ratio, $m^* \sim 2$) and very low structural damping (damping coefficient, $\zeta < 0.01$). In modeled riser motions, such as in the case of an elastically mounted circular cylinder, it has been shown that low structural damping and mass ratio can lead to large resonant amplitudes and increased bandwidth of the resonant region of excitation (Khalak and Williamson, 1996).

Very long, flexible risers exhibit structural characteristics similar to long beams or strings depending on the application. If one considers a long cylindrical structure to be similar to a beam under tension, it possesses a countable set of natural frequencies and modes in the in-line and transverse directions, as in equation 1, where n denotes mode number, E denotes modulus of elasticity, I is the area moment of inertia, L is the beam length, M is the mass per unit length, and T is the axial tension force in the beam.

$$f_{beam} = \sqrt{\frac{n^4 \pi^2 EI}{4L^4 M} + \frac{n^2 T}{4L^2 M}} \qquad (1)$$

The first term of equation 1 is the natural frequency of a particular mode for a bending dominated system, such as a beam, with simply supported end conditions. The second term of the equation is the natural frequency of a particular mode for a string in tension. Depending on the ratio of the tension to bending stiffness, the natural frequency of the n^{th} mode lies between n (string) and n^2 (beam) times the fundamental natural frequency of the system.

Vortex shedding in the wake of a fixed circular cylinder in a free stream is characterized by the Kàrmàn vortex street, where vortices shed periodically in the wake of the cylinder at the Strouhal frequency. For flexible cylinders or elastically mounted rigid cylinders, the wake behind the cylinder is characterized by the periodic shedding of vortices or groups of vortices, which similarly cause periodic forcing on the cylinder; forces exerted on the cylinder are a function of this periodic vortex shedding. The in-line excitation force from vortex shedding has a frequency twice the cross-flow forcing frequency as a result of the shedding process.

A condition of dual resonance may occur when the cross-flow natural frequency of the structure is equal, or at least close to the frequency of vortex shedding, f_s, and the in-line natural frequency is equal to twice the shedding frequency, i.e. $f_{ny} \sim f_s$ and $f_{nx}/f_{ny} = 2$. This condition is illustrated in Fig. 1, where the first mode of the tensioned beam is excited in the cross-flow (y) direction and the second mode is excited in the in-line (x) direction.

In a beam, the ratio f_{nx}/f_{ny} can rarely reach a value close to 2 because of the n^2 relation between beam natural frequencies. Cross-flow motion will resonate from fluid excitation in this case, while in-line motion is non-resonant. Shallow water structures are typically bending dominated systems, bearing characteristics closer to a beam than a string. These characteristics result in typical excitation of beam modes under the