

Computation of Slamming Forces on Wedges of Small Deadrise Angles Using a CIP Method

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ABSTRACT

This paper presents numerical solutions of the slamming problem for 2-D wedges of small deadrise angles entering calm water. The compressibility of air between the bottom of the wedge and the free surface is modeled. The highly nonlinear water entry problem governed by the Navier-Stokes equations was solved by a CIP based finite difference method on a fixed Cartesian grid. In the computation, a compact upwind scheme was employed for the advection calculations and a pressure-based algorithm was applied to treat the multiple phases. The free surface and the body boundaries were captured using density functions. For the pressure calculation, a Poisson-type equation was solved at each time step by the conjugate gradient iterative method.

Validation studies were carried out for water entry of wedges with various deadrise angles ranging from 0 to 10 degrees at constant vertical velocity. Computed pressures and air flows were compared with experimental data and the numerical solutions by other methods.

KEY WORDS: Navier-Stokes equations; CIP method; Slamming; Compressible air; Validation.

INTRODUCTION

When a ship travels in heavy seas, the large-amplitude ship motion can result in bow-flare water impact. It will subsequently cause severe damages to ship structures. The water-entry problem of wedge has been extensively studied by many researchers. The theoretical analysis of the similarity flow induced by the wedge entry was first conducted by Wagner (1932). Armand and Cointe (1986), Cointe (1991) and Howison et al. (1991) extended Wagner's theory to analyze the wedge entry problem using matched asymptotic expansions for wedges with small deadrise. Furthermore, Dobrovol'skaya (1969) developed an analytical solution in terms of a nonlinear singular integral equation for the problem of the symmetrical entry of a wedge into calm water. Based on the work of Vinje and Brevig (1981), Greenhow (1987) used Cauchy's formula to solve the wedge entry problem. In his work, both gravity and nonlinear free surface conditions were taken into account.

Zhao and Faltinsen (1993) studied the water entry of a wedge using boundary element method with constant elements. The jet tip at the intersection point of the body surface and the free surface was cut and two small constant elements were distributed. There are numerical difficulties to trace the water particles in the intersection point. Lin et al. (1984) presented an approach to treat the difficulties. In their work, the boundary integral equation derived from Cauchy's formula was discretized using linear elements so that the intersection points

can be used as the collocation points. Chuang et al. (2006) developed a boundary element method based on desingularized Cauchy's formula. A numerical approach was also developed to remove the corner singularity at the intersection point of body surface and free surface.

Although great progress has been made in computing the water-entry problem with potential-flow based methods, there are difficulties for these methods to treat highly distorted or breaking free surfaces, especially the compressed air when the deadrise angle of a wedge is close to zero degree. These difficulties can be overcome by the computational fluid dynamics (CFD) methods based on solving the Navier-Stokes equations. Recently, Kleefsman et al. (2005) have solved the 2-D slamming problem of symmetric bodies by the volume of fluid (VOF) method. Kim et al. (2007) used a smoothed particle hydrodynamics (SPH) method to simulate the water entry of asymmetric bodies. Zhu et al. (2005) studied water entry and exit of a horizontal circular cylinder using a finite-difference method based on the constrained interpolation profile (CIP) algorithm. The CIP method was proposed by Yabe et al. (2001) and further developed by Hu and Kashiwagi (2004) for violent free surface flow. Yang and Qiu (2007) applied the CIP-based finite difference method to compute symmetric and asymmetric water entry of wedges with various large deadrise angles. In most of these CFD methods, the compressible air layer between the solid surface and the free surface is neglected. However, for the water entry of wedges with very small deadrise angles, especially for the extreme case of a flat bottom, the air layer plays an important role and the compressibility of air needs to be considered.

In this paper, the CIP method is extended to solve the water-entry problem of wedges with small deadrise angles. A unified algorithm is applied to consider compressible and incompressible fluids. Validation studies were carried out for wedges with deadrise angles from 0° to 10°. Pressures, free surface elevations and air flows are compared with experimental data and solutions by other methods.

MATHEMATICAL FORMULATION

It is assumed that the fluid is compressible and viscous, and the temperature variations are neglected. As described in the work of Hu and Kashiwagi (2004), the governing equations for the fluid are given as

$$\frac{\partial \rho}{\partial t} + u_i \frac{\partial \rho}{\partial x_i} = -\rho \frac{\partial u_i}{\partial x_i} \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_j} + f_i \quad (2)$$