A Parabolic Equation for Wave Propagation over Porous Media

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ABSTRACT

A numerical wave model is developed to simulate wave transformation in the surf zone. The governing equation is the parabolic formulation of the mild-slope equation including wave breaking and energy dissipation over porous media. Following Mordane et al. (2004) the present theory is obtained by splitting the Berkhoff’s equation operator into two parabolic operators representing progressive and reflected wave propagation, respectively. The use of the Padé \[2,2\] approximation permits to derive the parabolic equation for transmitted waves and to model wave propagation at very wide propagation angles over a porous bed. The model is verified through experiments for waves propagating over a uniform and composite uniform impermeable slope and waves passing over submerged permeable breakwaters with porosities. The parabolic model is also used for the case of wave propagation at large angle incidence over a porous elliptic shoal. Energy dissipation due to wave breaking has been accounted for in the computations. Numerical results are in good agreement with experimental results.

KEYWORDS: parabolic; mild-slope equation; porous media; wave breaking; Padé approximation.

INTRODUCTION

The porous structures, such as seawalls, detached breakwaters or submerged breakwaters, are frequently used to protect coastal erosion from wave attack. The effectiveness of these structures is owing to that they are able to reflect and absorb wave energy and to dissipate it by wave breaking on the structures. Wave energy reduction on the lee side of the porous structures increases so that only a small part of the wave energy is transmitted to onshore. Consequently, the wave field in the lee side of the structures becomes quiet and the intensity of the wave action on the shoreline decreases, and the migration of coastal erosion and its corresponding coastal disasters can be protected. The knowledge of wave transformation over porous media is required to determine the stability of the porous media and to evaluate the effectiveness of wave energy reduction of the structures.

Several numerical models have been developed to study wave transformations over porous structures. Based on the mild-slope assumption, Rojanakamthorn et al. (1989) adopted Sollitt and Cross (1972) theory to derive a modified mild-slope equation (MSE) for describing non-breaking waves traveling over a general finite porous bed. The MSE is suitable for an unsteady flow in a porous medium which is linearized using an approximation of the nonlinear friction forces. The wave energy dissipation due to resistance in the porous structures is considered in their model and the depth-averaged equation yields an elliptic-type MSE on permeable bed. However the amount of computer memory and calculation required render the elliptic equation inefficient for application to a large coastal area. Losada et al. (1996) presented similar studies of the kinematics and dynamics of wave interaction with permeable breakwaters under non-breaking oblique incident regular waves and directional random waves. The influence of structure geometry, porous material properties, and wave characteristics over and inside the breakwaters was also investigated.

The solution of MSE requires a computational effort which may become too large when large coastal areas have to be considered. As with advanced computer technology, engineers also want to deal with larger coastal regions, which is still needed to develop a simple model to be applied in those situations. Therefore, it is desirable to develop a simpler model that can provide a trustworthy results to the practical applications. A parabolic approximation of MSE is known to be a good one of this alternative (Hsu and Wen, 2001). A detailed discussion of the parabolisation is referred to the text book of Dingemans (1997) and the paper of Hsu and Wen (2001).

For this reason a time-dependent parabolic type MSE is selected to simulate wave filed in the vicinity of porous media. For instance, Radder (1979) transformed the Berkhoff’s elliptic MSE into a Helmholtz equation using a scaling factor in which the operator of this equation is split into a product of two parabolic operators, the first one representing reflected waves (back forward waves) and the second one representing forward scattering waves (progressive waves). Generally, the parabolic approximation consists in neglecting the backward waves.