A numerical solution of the 3D irrotational Navier-Stokes equations for non hydrostatic wave propagation

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ABSTRACT

A numerical solution of the 3D Navier-Stokes equations is developed for irrotational flow by introducing a new set of unknowns in the equations making them amenable to a numerical analysis by using the method of characteristics. The efficiency and accuracy of the developed numerical model is successfully validated against a number of benchmark tests including propagation of a second order Stokes wave along a channel, formation of a nonlinear standing wave inside a basin, propagation of a solitary wave over a constant bed and wave reflection and transmission over a plane bottom slope.

KEY WORDS: Non linear waves; non hydrostatic flow; wave propagation; three-dimensional numerical solution; Navier-Stokes equations; method of characteristics; solitary waves.

INTRODUCTION

Water wave propagation and its interaction with uneven bottom, traveling currents, floating bodies, breakwaters and other flow obstacles etc. is an important matter in many applications of ocean and coastal engineering. Therefore, it is not surprising that over the years, a great amount of work has been devoted to methods solving numerically the complex equations describing the phenomenon. In its general form, wave propagation is described by the complete Navier-Stokes equations (NSE) with the inclusion of all non linear terms, viscous and rotational flow effects and non hydrostatic pressure at the surface and within the fluid.

While three dimensional NSE based hydrostatic models have been successfully operating for some decades now, for example, in the field of estuarine and coastal flow, it is only recently that research effort has been directed towards the development of non hydrostatic NSE models, mainly, because these models have nowadays become computationally affordable. In the interim period, a reliable intermediate has been developed and applied successfully, despite its computational cost limitations in complicated situations of concern, in the form of depth-integrated, either of the mild slope equation, Boussinesq type or highly nonlinear and dispersive models (for a review, Liu and Losada (2002)). Recently, Athanassoulis and Belibassakis (2002) have successfully generalized this kind of approach by introducing a nonlinear coupled-mode method that adds two extra modes in the series expansion of wave potential and ensures consistency and accuracy of the results without resorting to any mild slope assumptions.

One of the main issues concerning the non hydrostatic NSE simulation of a free surface flow, is the numerical description of the moving boundary at the air-water interface. Several methods have been applied as part of the solving procedure such as the arbitrary Lagrangian-Eulerian (ALE) method (Hodges and Street, 1999; Zhou and Stansby, 1999), the marker and cell (MAC) method (Park et al. 1999), the volume of fluid (VOF) method (Ng and Kot, 1992; Shen et al. 2004; Nielsen and Mayer, 2004) and the level-set method (Iafrati and Campana, 2003; Yue et al. 2004). It is true that these methods can successfully deal with complicated free surface boundaries such as a breaking wave but the computational cost is high and finally, the strict requirements imposed on stability reduce substantially their range of practical application.

On the other hand, in the case of free surface elevations that may be considered as single-valued functions of the horizontal position (e.g. pre-breaking waves), NSE models were possible to be developed which are capable of tracking surface motions at a relatively smaller computational cost. In this regard, three numerical methods of solution have been reported (see Choi and Wu, 2006): (a) the explicit projection method (Chorin, 1968; Li and Fleming, 2001; Lin and Li, 2002), the semi-implicit, fractional step method (Casulli, 1999; Casulli and Zanolli, 2002; Chen, 2003) and (c) the implicit method (Namin et al. 2001; Yuan and Wu, 2004). The two first methods lead to a Poisson type equation for pressure calculation (PPE) which is solved by an iterative matrix solver, while in the implicit method, the resulting matrix containing the unknown horizontal velocities, is solved in a direct manner. In all cases, a large number of layers along the vertical (ten to twenty) need to be used, if the flow characteristics are to be calculated at an acceptable accuracy.