Influence on Membrane Stresses in a Thin Shell Due to a Hole

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ABSTRACT

A numerical method to find the influence on the membrane stresses in a shell due to a hole is proposed. Details for the case of a spherical shell having an elliptical hole are thoroughly presented. The results for a typical case are given. It is found that the influence is of localized nature as perceived.

KEY WORDS: Membrane stress, shell hole, spherical shell, numerical procedure, finite difference method

INTRODUCTION

Shell structures have been extensively used in civil, aerospace, nuclear, marine, offshore, and petrochemical industries. In many practical applications, there is a need to make a small hole in a shell. For instance, a hole must be made in the shell as a chimney goes through a shell roof, an elliptic manhole breaks the continuity of a closed vessel or a circular pipe is installed to a tank. Although the cross sections of the chimney, manhole, and pipe are well defined, their intersections with the shells, or hole edges, are three dimensional closed curves. For convenience, let the longitudinal axes of chimney, manhole, pipe, etc., passing through the centroids of the cross sectional areas be defined as the hole axes, and the cross sectional shapes be named as the hole shapes.

Finding the influence of a hole on the membrane stress resultants in the vicinity of the hole is the objective of this study. The superposition principle is utilized. First, assume that the membrane stress resultants of the shell without the hole due to the external loading are already known. Next, consider equal but opposite stress resultants as the only loads acting on the shell along the hole edge, and the membrane stress resultants in the vicinity of the hole due to these edge loads are computed. Finally, by superimposing the stress resultants from the preceding two steps, the membrane stress resultants of the shell with the hole due to the external loads are obtained.

In this paper, a systematic procedure to find the membrane stresses in the vicinity of the hole due to the applied edge loads is presented. The hole considered has an elliptic shape. The equilibrium equations and the edge loads constitute an initial-value problem. It is solved by the finite difference method. A spherical storage tank with an elliptic hole subject to a uniform internal pressure is thoroughly studied. An elliptic-hyperbolic coordinate system is used. A computer program was written to do the numerical computations. The results of a typical example are given.

Coordinate Systems

As shown in Fig. 1, a point on the spherical shell surface with a radius $R$ is given by the two parameters $x'$ and $y'$; the colatitude and longitude. With reference to the three dimensional $X'Y'Z'$ rectangular coordinate system, CSI, the surface is defined by

\[
\begin{align*}
X' &= R \sin x' \cos y' \\
Y' &= R \sin x' \sin y' \\
Z' &= R \cos x'
\end{align*}
\]  

As the projection of the hole on a given plane is an ellipse, it is convenient to define the ellipse in the given plane as follows (Fig. 2). The $X$ axis follows the major axis and $Y$ follows the minor axis. The semi-major axis, $a$, and the semi-minor axis, $b$, can be expressed in another two constants $C$ and $x_i$, such that

\[
a = C \cosh x_i \\
b = C \sinh x_i
\]  

Or,

\[
C = \sqrt{a^2 - b^2} \\
x_i = \tanh^{-1} \frac{b}{a}
\]  

Associated with this ellipse, a planar elliptic-hyperbolic coordinate system, Fig. 2, is introduced as follows

\[
\begin{align*}
X &= C \cosh x \cos y \\
Y &= C \sinh x \sin y
\end{align*}
\]  

Thus, the equation of the ellipse is parametrized in $y$ by setting $x = x_i$, $m$ constant in the above two equations. Also, it should be noted that the $y$-coordinate lines are elliptic curves around the hole projection and the $x$ coordinate lines are hyperbolic curves normal to the $y$ coordinate lines. Consider a surface embedded in the three dimensional $X, Y, Z$ space. By