A New Model on Equilibrium Spectrum of Wind Waves

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ABSTRACT

Based on the joint-distribution of wave amplitude and wave period, a new model on the equilibrium spectrum of wind waves is derived. The derived model is related to the wind speed, spectral width and wave age. The related coefficients are determined empirically based on the observation. This model can be used to explain the variability of spectral level and spectral curve slope. It is found that the spectral level is controlled by both wind speed and wave age; the slope of spectral curve is related to only the wave age. The relationships between the spectral width and wave age, between the spectral level and wave age are obtained based on the spectral properties and empirical observations.

KEY WORDS: waves, spectrum, equilibrium, wave age, spectral width.

1 INTRODUCTION

Generally, the "equilibrium spectrum" is limited to the high frequency (or wavenumber) tail rather than the entire frequency (or wavenumber) range. The first model of equilibrium frequency spectrum for deep water waves was proposed by Phillips (1958) based on dimensional analysis as

$$ S(\omega) = \alpha g^2 \omega^{-5}. $$

(1)

The corresponding unidirectional wavenumber spectrum is

$$ S(k) = \frac{\alpha}{2} k^{-4}, $$

(2)

where $\alpha$ is the equilibrium constant, $g$ is the gravitational acceleration, $\omega$ is the angular frequency, $k$ is the wavenumber.

For the frequency region $1.5 \omega_p < \omega < 3.5 \omega_p$ and the wave age region $0.83 < \frac{U_1}{C_p} < 5$, Donelan et al. (1985) obtained

$$ S(\omega) = 0.006 \left( \frac{U_1}{C_p} \right)^{0.55} (\frac{\omega}{\omega_p})^{-5} \exp\left(-\frac{\omega^4}{\omega_p^4}\right) \gamma^\Gamma $$

(3)

from the observations of water surface elevation using 14 wave staffs in an array in Lake Ontario and in a large laboratory tank. Here $\omega_p$ is the angular frequency at the spectral peak, $C_p$ is the phase speed of the waves at spectral peak, $\gamma^\Gamma$ is the peak enhancement (over the Pierson-Moscowitz spectrum) factor.

In this paper, we will derive the equilibrium spectrum of wind waves, based on both dynamical and statistical considerations. At first, the wind energy is input through the work done by air pressure component induced by large-scale water waves. In smaller scale, the wind energy can be directly input through the wind friction stress. The two types of wind input may overlap in a part of the equilibrium range. Therefore, the measured equilibrium spectra of wind waves should include both contributions from the two mechanisms. The equilibrium spectrum derived in this paper represents the energy density contributed by the first mechanism. The spectrum of wind-induced gravity-capillary waves (Liu 1996; Liu and Yan 1995) may represent the energy density contributed by the second mechanism. The sum of the above two spectra may represent the energy density in the equilibrium range, especially for higher frequency range and stronger wind condition. In the statistical consideration, a model of equilibrium spectrum will be derived from the joint-distribution of wave amplitude and wave period of gravity waves. In other words, the model is derived from theories and observations on the statistical property of wave characteristic variables,