Link-Beam Model for Dynamic Buckle Propagation in Pipelines

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Abstract
A model for dynamic buckle propagation is proposed. The cornerstone of the model is the equilibrium of a collapsing elasto-plastic ring under constant pressure and concentrated forces. The pipeline is idealized as individually collapsing rings linked by a couple of longitudinal beams. The force interaction between the link-beams and the rings is described by closed form equations. This approach allows the longitudinal bending work to be calculated separately from the cross section work. Approximations of the pipe kinetic energy and the energy dissipated through the medium during dynamic buckle propagation are proposed. Predictions of the propagation pressure vs. the velocity of propagation for pipes in vacuum and water are within engineering agreement to experimental results. Therefore, a novel description of buckle propagation is developed, leading to an improved understanding of the problem.

Introduction
If a dent occurs in a section of a deepwater pipeline, the stiffness of the damaged region is decreased. The external pressure might be sufficient to collapse the pipe cross section such that two opposite interior surfaces come into contact. The damage then spreads longitudinally attaining a characteristic profile of propagation within a few diameters, Fig. 1. This type of failure is known as the propagating buckle. The minimum pressure necessary to sustain propagation is known as the propagation pressure, \( P_p \).

The propagation pressure can be obtained under volume controlled experiments which simulate quasistatic propagation. Quasistatic buckle propagation has been extensively researched both experimentally (Kyriakides 1994) and analytically (Nogueira 1993, Nogueira and Tassoulas 1995). However, few studies have dealt with dynamic propagation. In this regard, velocities of propagation of the order of 150 m per second have been reported by Mesloh et al. (1976) for external pressures slightly above \( P_p \). Kyriakides and Babcock (1979) reported experimental results relating the velocity of propagation to the external pressure for buckles propagating in pipes in air and water. Song and Tassoulas (1990) used an elasto-plastic finite element formulation to model dynamic buckle propagation. This paper briefly describes a recently developed model (Nogueira 1998) for quasistatic buckle propagation and extends it to dynamic propagation by means of balance energy which includes estimates of the pipe kinetic energy and the energy dissipated through the medium during propagation.

Fig. 1 shows a computer generated half pipe in which a buckle is propagating. The axes \( x_1, x_2, x_3 \) are defined in Fig. 1. The tail of the propagating buckle is the collapsed region where contact takes place. The front of the buckle is the undeformed region towards which the buckle is propagating. The propagating profile is the transition region that joins the front to the tail. Its trace in the plane \( x_2 = 0 \) (that is, the longitudinal fiber at the 12 o’clock position) is the top generator.

Cross-section collapse
Palmer and Martin (1975) presented the first analytical model for quasistatic buckle propagation. They idealized the material as rigid-perfectly plastic and assumed that all cross sections of a pipe collapse as a ring during buckle propagation, with all deformation restricted to four hinges, Fig. 2. The internal work dissipated by the ring as the cross section collapses is \( W^R = \frac{\pi}{2} R \sigma_0 \rho^2 \). The external work is the product of the (constant) propagation pressure and the cross sectional area decrease \( \Delta S \), \( W^E = P_p \Delta S \). Using \( W^R = W^E \), and defining \( \sigma_0 = \) material plastic stress, \( t = \) pipe thickness, \( R = \) outside pipe radius, \( D = 2R \), \( \Delta S = D^2/2 \), the following estimate of the propagation pressure is obtained:

\[
P_p = \pi \sigma_0 \left( \frac{I}{D} \right)^2
\]

This approximation of the propagation pressure takes into account only the work done by cross-section bending. In order to estimate the work done in bending of longitudinal fibers, a model which links the ring (cross-section) collapse to longitudinal bending through an equilibrium condition is developed below. To this end, a force \( F \) acting along the \( x_3 \) axis is introduced on the collapsing rigid-perfectly-plastic ring (of unit length). All the following equations describe a force \( F \) applied on a quarter ring as shown in Fig. 3. Due to the double symmetry of the problem, at \( x_3 = 0 \) the reaction \( R_h \) is parallel to \( x_2 \). At \( x_3 = 0 \) the reaction \( R_v \) is parallel to \( x_3 \). The plastic moment \( M_p \) acts at both locations. An external pressure \( P_p \) acts on the ring, which leads to vertical force \( V \) and horizontal force \( H \), see Fig. 3 for several definitions. Equilibrium of forces leads to \( R_h = H \) and \( R_v = V + F \). Using that the plastic moment \( M_p = \sigma_0 f^2 A_0 \), moment equilibrium yields: