Wave Forces on a Circular Island Using Perturbation-DRBEM

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ABSTRACT

In this paper, an effective numerical model using perturbation technique and the dual reciprocity boundary element method (DRBEM) is presented to evaluate wave forces on a circular island over a varying topography. The varying depth is treated as a perturbed constant depth. Using perturbation technique on the mild-slope equation, one can obtain an ordinary Helmholtz equation for constant water depth and a corresponding inhomogeneous Helmholtz equation for the variable water depth. The inhomogeneous part will result in a domain integral by applying the conventional boundary element method. It will make the computation complicated and messy. To improve this drawback, the DRBEM is adopted to transform the domain integral into boundary integral. To verify the validity of this model, the relative amplitude around a circular island was calculated, and compared with analytical solutions of Homma(1950). The agreement was fairly satisfactory. The wave forces on a circular island over a seabed with a variable depth were further calculated by this model.

KEY WORDS: Wave forces, Perturbation, DRBEM.

INTRODUCTION

Wave loading on large structures has been the subject of intensive research over the past few decades. With increasing engineering activities in both coastal and offshore regions nowadays, accurate prediction of wave forces on large structures has become progressively important.

The response of water waves around an island has been studied intensively. Homma(1950) proposed an exact solution of wave response by using shallow water wave approximation. To model the refraction and diffraction of water waves around ocean structures, Berkhoff(1972) introduced the well known mild-slope equation that simplified the three dimensional problems to a two dimensional one. Later Smith and Sprinks(1975) gave a formal derivation of the mild-slope equation. Based on the mild-slope equation, many numerical models have been developed by adopting hybrid-element method to calculate wave field in the vicinity of an island (Jonsson et al.(1976), Bettess and Zienkiewicz(1977), Houston(1981), Tsay and Liu(1983)). The hybrid-element method performs well on varying water depth, but it still needs to place with a large number of finite elements in variable seabed domain, and hence the size of the coefficient matrix become very large. The finite difference method has been used for numerical approach on the mild-slope equation, for e.g. Madsen and Larsen (1987), Panchage and Pearce(1991). Li and Anastasiou(1992) proposed a multigrid technique, which appeared to be much more efficient in comparison with other finite difference method. However, all the finite difference models have a common drawback, the boundary of a computational domain cannot be realistically modeled, as a results of the stair-step representation in the finite difference method. The boundary element method (BEM) only requires a discretization on the boundary of a computational domain and it has been widely applied in engineering practice. Au and Brebbia(1983) used BEM to solve the diffraction of waves around vertical cylinders with constant water depth and obtained excellent results. However, when the water depth becomes a variable, a domain integral arises and the domain has to be discretized, considerably increasing the number of unknowns. To avoid discretization in the domain, Rangogni(1988) combined perturbation method with the boundary element method. The varying depth is treated as a perturbed constant depth and in this way the domain integral is reduced to an entirely known function. This model seems incomplete, because it still has a domain integral to be dealt with. To overcome the difficulties associated with singular domain integrals, the DRBEM (duel reciprocity boundary element method) was first proposed by Nardini and Brebbia(1982) and was later extended and improved by many authors. Zhu(1993a) used the DRBEM with great success to reduce the domain integral to boundary integral, but an increase in the number of unknowns compared with the constant depth solution is not avoided. In this paper, a combined perturbation method with the DRBEM model is proposed to improve the deficiency in the model proposed by Rangogni(1988) and Zhu(1993a).

FORMULATION

Consider the case that incident waves of angular frequency, \( \omega \), travelling in the positive \( x \) direction are diffracted and refracted by a