Performance Enhancement of the Oscillating Wave Surge Converter by a Breakwater

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The present study is motivated by recent results showing that an Oscillating Wave Surge Converter (OWSC) located near a straight coast can achieve much higher levels of efficiency than one in the open ocean. The aim is to analyse, using a three-dimensional mathematical model, whether a breakwater of finite width can replicate such performance features. Results show that a breakwater can significantly enhance the performance of an OWSC when located very close to it. A random sea analysis reveals that the new OWSC-breakwater hybrid model can be highly effective in locations with high occurrence of low sea states.

INTRODUCTION

A plethora of wave energy conversion systems are now known in the literature, and the prototypes of a few are already tested. The race to produce commercially viable ocean wave power is still an open challenge though. Some concepts have emerged to be promising, and the OWSC is one of them. The OWSC has a wide bandwidth of consistent performance levels, which has generated a lot of interest in such systems. It is a buoyant flap-type shallow water WEC, capturing energy by virtue of its pitching motion and is ideally located in water depths of 10–15 m. Many research efforts are now focused on identifying mechanisms to enhance the performance and address the shortcomings of the existing design (see, e.g., Sarkar, Doherty, and Dias, 2014). The recent study by Sarkar, Renzi, and Dias (2013a) showed that a straight coast has an “oxymoronic” kind of influence on the OWSC performance. However, when the flap is located close to the straight coast, it can consistently capture much higher levels of energy. The present study tries to investigate whether a breakwater present on the lee-side of the OWSC can realize such predominantly favourable effects so that it can further enhance the performance of the device. The two-dimensional (2D) analysis of a similar system comprised of two flaps was investigated in Srokosz and Evans (1979), and they found that a combination of two flaps can result in higher efficiency. In fact, in the 2D analysis, the latter showed that maximum efficiency up to 1 is possible, which corroborates the maximum efficiency obtained for a 2D analysis of a single OWSC located near a straight coast (Sarkar, Renzi, and Dias, 2013a). However, in the 3D description of the straight coast problem, much higher levels of efficiency are shown to be possible (see Sarkar, Renzi, and Dias, 2013a, 2014a).

The goal of this study is to investigate whether it is possible to artificially produce hydrodynamic characteristics similar to those of the straight coast via the introduction of a breakwater.

The study is also important because the presence of coastal structures generally can influence the performance of the OWSC. Breakwaters are installed in coastal regions for various reasons. Construction of the breakwater by itself is a costly investment. With the implementation of this hybrid form of system, one incurs an additional expenditure, but then there is the possibility of recovery, which may ensure the economic viability of such projects. Some research studies in the literature have proposed hybrid systems that can serve both coastal protection and energy generation. For example, Schoolderman et al. (2011) presented a WEC integrated with a caisson breakwater, where the dynamic wave pressure exerted on the underwater opening on the front side of the breakwater produces a flow of water into a ramp with gradual constriction, then through a turbine located at the rear, and finally into the sea.

Although it may be stated that a very long breakwater would be like a straight coast and therefore result in similar hydrodynamics of the OWSC as those near a straight coast, it is important to quantify the physical dimensions of the breakwater. The analysis is also important for the economic perspective. So, if one tries to compromise on the width of the breakwater, then how does that impact the performance of the converter? Is it then wise to use a breakwater under such circumstances?

The present study extends a recently developed mathematical model, which has been extensively used in the analysis of the OWSC in various circumstances (see, e.g., Renzi and Dias, 2012; Sarkar, Renzi, and Dias, 2014b, 2014c). The formulation is based on Green’s integral equation, Green’s function, yielding an expression of the velocity potential in terms of the jump in potential across the two sides of the OWSC and the breakwater. The unknown jumps in the potential are then expanded in terms of the Chebyshev polynomial to solve the hypersingular integrals resulting from the kinematic boundary conditions on the OWSC and the breakwater. The solutions are then used to evaluate the hydrodynamic parameters and performance of the OWSC. In the following section, the mathematical model is presented, and thereafter the behaviour of the system in regular and irregular waves is analysed. Comparison is also made with the hydrodynamics and performance of the OWSC near a straight coast and in the open ocean.

MATHEMATICAL MODEL

The OWSC is located in an ocean of constant water depth $h'$ with the breakwater located at a distance $d'$ behind the flap. Waves of amplitude $A_j'$ are incident from the right, forming an angle of $\psi$ with the x-axis, as shown in Fig. 1. The origin is located at the center of the mean free surface on the flap with $x'$ pointing away from the breakwater and the flap and $z'$ directed upwards. The center of the flap and breakwater are oriented along the same x coordinate.

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The velocity potential satisfies the Laplace equation:

\[ \nabla^2 \Phi' = 0 \]  

in the fluid domain, where \( \nabla f' = (f'_x, f'_y, f'_z) \) is the nabla operator; the subscripts with commas denote differentiation with respect to relevant variables. The linearised kinematic-dynamic boundary condition on the free surface gives:

\[ \Phi_{,zz'} + g \Phi_{,z} = 0, \quad z' = 0 \]  

where \( g \) is the acceleration due to gravity. The no-flux condition at the sea bed gives:

\[ \Phi_{,z} = 0, \quad z' = -h' \]  

Lastly, the kinematic condition on the lateral surfaces of the flap is expressed as:

\[ \Phi_{,y} = -\theta, (z' + h' - c')H(z' + h' - c'), \quad x' = 0 \pm \varepsilon', \quad \varepsilon' \rightarrow 0, \quad |y'| < \frac{w'}{2} \]  

and that on the breakwater as:

\[ \Phi_{,y} = 0, \quad x' = -d' \pm \varepsilon', \quad \varepsilon' \rightarrow 0, \quad |y'| < \frac{b'}{2} \]  

using the thin plate approximation. Consider the following nondimensional system of physical variables:

\[ (x, y, z, d, b) = (x', y', z', d', b')/w', \quad t = \frac{R}{w' \sqrt{g}}, \quad \Phi = \frac{\Phi'}{\sqrt{g w' A'}}, \quad \theta = \theta \]  

where \( \varepsilon = w'/A' \) is the small parameter for the problem. Separating out the time dependence from the unknown system of variables with the assumption of simple harmonic oscillation gives:

\[ \left\{ \begin{array}{c} \Theta \\ \Phi \end{array} \right\} = \text{Re} \left( \left\{ \begin{array}{c} \phi'(x, y, z) + \phi''(x, y, z) + V \phi^R(x, y, z) \\ \phi''(x, y, z) + \phi^R(x, y, z) \end{array} \right\} e^{-i\omega t} \right) \]  

where \( V = \text{Im} \Theta, \omega = \sqrt{\sqrt{g} w'/R} \) and \( \Theta \) are the angular frequency and amplitude of oscillation of the flap, respectively, while \( \phi'(x, y, z), \phi''(x, y, z), \) and \( \phi^R(x, y, z) \) are the complex spatial incident, radiation, and scattering potential, respectively. The incident wave potential is a known parameter for the problem and is defined as:

\[ \phi'(x, y, z) = \frac{i A \cosh k (z + h)}{\cosh kh} \exp\left(-i k \cos \phi + i k y \sin \phi \right) \]  

where \( \phi \) is the angle of incidence with respect to the negative \( x \) axis, as shown in Fig. 1. The unknown potentials can be further simplified by separating out the vertical dependence in the form of normalised vertical eigenmodes (see Mei et al., 2005):

\[ Z_n(z) = \frac{\sqrt{2} \cosh \kappa_n (z + h)}{(h + \omega^2 \sinh \kappa_n h)^{1/2}} \]  

which yields:

\[ \phi^{(R,D)}(x, y, z) = \sum_{n=0}^\infty \phi_n^{(R,D)}(x, y) Z_n(z) \]  

In Eq. 9, \( \kappa_0 = k \) and \( \kappa_n = i k_n \) are the solutions of the dispersion relation:

\[ \omega^2 = k \tanh kh, \quad \omega^2 = -k_n \tan k_n h, \quad n = 1, 2, \ldots \]  

respectively. The vertical eigenmodes satisfy the orthogonality relation:

\[ \int_{-h}^0 Z_n(z) Z_m(z) dz = \delta_{nm} \]  

where \( \delta_{nm} \) is the Kronecker delta and also ensures that the linearised free surface boundary condition (Eq. 2) and no-flux boundary condition at the sea bed (Eq. 3) are automatically satisfied.

Application of the factorisation (Eqs. 7 and 10) and using the orthogonality relation (Eq. 12) yield a two-dimensional boundary-value problem in terms of the spatial radiation and diffraction potentials, where the Laplace equation (Eq. 1) becomes the Helmholtz equation:

\[ (\nabla^2 + \kappa_n^2) \left\{ \begin{array}{c} \phi_n^R \\ \phi_n^D \end{array} \right\} = 0 \]  

and the kinematic condition on the flap (Eq. 4) becomes:

\[ \left\{ \begin{array}{c} \phi_n^R \\ \phi_n^D \\ \phi_n^D \\ \phi_n^D \end{array} \right\} = \left\{ \begin{array}{c} f_{n0} \delta_{nm} \\ A_i d_m e^{i k y \sin \phi} \end{array} \right\} \quad x = x_m \pm \varepsilon, \quad \varepsilon \rightarrow 0, \quad -\frac{w}{2} < y < \frac{w}{2} \]
where

\[ f_n = \frac{\sqrt{2} \left[ \kappa_n (h - c) \sinh (\kappa_n h) + \cosh (\kappa_n c) - \cosh (\kappa_n h) \right]}{\kappa_n^2 (h + \omega^2 \sinh^2 (\kappa_n h))^{1/2}} \]  \hspace{1cm} (15)

and

\[ d_{n_1} = - \frac{k \cos \psi (h + \omega^2 \sin^2 k h) \delta_{0,n}}{\sqrt{2 \omega} \cosh k \delta_{0,n}} \]  \hspace{1cm} (16)

are constants depending on the geometry of the system. Similarly, the kinematic condition on the breakwater (Eq. 5) yields:

\[
\begin{pmatrix}
\psi_{n_1}^x \\
\psi_{n_1}^z
\end{pmatrix} = \begin{pmatrix}
0 \\
A_1 d_{n_2} e^{i y \sin \phi}
\end{pmatrix}
\]

\[ x = -d \pm e, \ e \to 0, \]

\[ -\frac{b}{2} < |y| < \frac{b}{2} \]  \hspace{1cm} (17)

with

\[ d_{n_2} = - \frac{k \cos \psi (h + \omega^2 \sin^2 k h) \delta_{0,n}}{\sqrt{2 \omega} \cosh k \delta_{0,n}} \times \left( \cos (k d \cos \psi) + i \sin (k d \cos \psi) \right) \delta_{0,n} \]  \hspace{1cm} (18)

Lastly, both \( \phi^R \) and \( \phi^D \) are required to be outgoing disturbances of the wave field (Mei et al., 2005). The above system of equations (Eqs. 13, 14, and 17) is then solved by using the procedure outlined in Renzi and Dias (2012) with the application of Green’s integral theorem (see Appendix for the solution details). Finally, solutions for the spatial radiation and diffraction potentials are obtained as:

\[ \phi^R(x, y, z) = \frac{i}{8} \sum_{n_1} \kappa_{n_1} Z_{n_1} (z) w \sum_{p_{n_1}} \int_{-\infty}^{\infty} \left( 1 - u^2 \right)^{1/2} U_p (u) \bigg\{ a_{p_1} H^{(1)}_{1} (\kappa_{n_1} \sqrt{x^2 + (y - uw/2)^2}) \bigg/ \sqrt{x^2 + (y - uw/2)^2} \bigg\} \]  \hspace{1cm} (19)

and

\[ \phi^D(x, y, z) = \frac{i}{8} A_1 k Z_0 (z) w \sum_{p_{n_1}} \int_{-\infty}^{\infty} \left( 1 - u^2 \right)^{1/2} U_p (u) \bigg\{ b_{p_1} H^{(1)}_{1} (\kappa_{n_1} \sqrt{x^2 + (y - uw/2)^2}) \bigg/ \sqrt{x^2 + (y - uw/2)^2} \bigg\} \]  \hspace{1cm} (20)

respectively, where \( H^{(1)}_{1} \) is the Hankel function of the first kind and first order; \( U_p \) is the Chebyshev polynomial of the second kind and order \( p, p = 0, 1, . . . ; P \in \mathbb{N} \), \( a_{p_1} \), \( a_{p_2} \), \( b_{p_1} \), and \( b_{p_2} \) are the complex solutions of a system of equations, which are solved numerically by using a collocation scheme (see Appendix for details). The expressions for the radiation and diffraction potential given by Eq. 19 and Eq. 20, respectively, have a similar form to solutions obtained for the same potentials in the other recently solved problems, e.g., a flap in an open ocean (Renzi and Dias, 2013), channel (Renzi and Dias, 2012), and near a straight coast (Sarkar, Renzi, and Dias, 2014a). The above solutions for the potentials are then utilised to solve the body equation of motion of the flap.

HYDRODYNAMIC PARAMETERS

While the breakwater is fixed, the bottom hinged flap undergoes pitching motion about its mean vertical position due to the pressure difference across its two sides. The nondimensional equation of motion of the flap is expressed as:

\[ [-\omega^2 I + C - i \omega \nu_{pt}] \Theta = i \omega \int_{-h}^{h} \Delta \phi (z + h - c) \ dy \ dz \]  \hspace{1cm} (21)

where \( I = I' / (\rho u'^2) \) is the second moment of inertia of the flap, \( C = C' / (\rho u'^3) \) is the coefficient of the net restoring flap buoyancy torque, \( \nu_{pt} = \nu_{pt} / (\rho u'^3 \sqrt{g/w}) \) is the power take-off (PTO) damping coefficient, and

\[ \Delta \phi = \phi (0 - e, y, z) - \phi (0 + e, y, z), \ e \to 0 \]  \hspace{1cm} (22)

is the difference in potential across the two sides of the flap. In Eq. 21, it is assumed that the power take-off damping is linearly proportional to the velocity of the flap oscillations. Decomposing the complex hydrodynamic torque due to the radiation potential into real and imaginary components, and using Eqs. 10, 35, and 40 transform Eq. 21 into:

\[ [-\omega^2 (I + \mu) + C - i \omega (\nu + \nu_{pt})] \Theta = F_1 \]  \hspace{1cm} (23)

where

\[ \mu = \frac{\pi w}{4} \text{Re} \left\{ \sum_{n_0} f_n a_{n_01} \right\}, \quad \nu = \frac{\pi \omega w}{4} \text{Im} \left\{ \sum_{n_0} f_n a_{n_01} \right\}, \quad F_1 = - \frac{\pi \omega w}{4} i A_1 b_{n_01} f_0 \]  \hspace{1cm} (24)

are the added inertia, radiation damping, and excitation torque due to the pitching motion of the flap, respectively. The converter captures maximum power for optimal damping of the power take-off system:

\[ \nu_{pt} = \sqrt{\frac{(C - (I + \mu) \omega^2)^2}{\omega^2} + \mu^2} \]  \hspace{1cm} (25)

and the optimal captured power is expressed as:

\[ P_{opt} = \frac{|F|^2}{4(\nu_{pt} + \nu)} \]  \hspace{1cm} (26)

which is a well-known relation for isolated wave energy converters (Falnes, 2002).

The performance of the converter is described in terms of capture factor, which is defined as the ratio of the absorbed power per unit width of the OWSC to the incident wave power per unit crest width, i.e.:

\[ C_F = \frac{P_{opt}}{\frac{1}{2} C_g A_t^2} \]  \hspace{1cm} (27)

where \( C_g \) is the well-known group velocity of the incident waves. In the next section, computations are performed for various widths of the breakwater \( b' \) and the distance of separation \( d' \) for a fixed OWSC configuration that resembles the WEC Oyster developed by Aquamarine Power with \( w' = 26 \text{ m}, \ c' = 4 \text{ m}, \) and the water depth \( h' = 13 \text{ m}. \)
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RESULTS

Figure 2 plots the variation of excitation torque versus the nondimensional parameter $k'd'$ for $d'=50$ m. Comparison is made with the behaviour of the OWSC near a straight coast for the same distance of separation (black solid line).

Figure 3 shows the variation of the hydrodynamic parameters of the OWSC: (a) added inertia $\mu'$ and (b) radiation damping $\nu'$ versus the nondimensional parameter $k'd'$. For a breakwater of width half that of the flap, the behaviour of the radiation coefficients is similar to that of a single flap in the open ocean. However, the formation of spikes in the radiation coefficients is observed near $k'd' = (m+1/2)\pi$, $m = 1, 2, \ldots$ as the breakwater width increases. The occurrence of the resonances in the straight coast problem has been explained by showing an analogy of the system to the anti-symmetric motion of a plate in a channel aligned with its centreline. The frequencies for $k'd' = (m+1/2)\pi$ actually correspond to the natural sloshing modes in a channel.

Thus, in general, we find that the hydrodynamic behaviour of the OWSC with a small breakwater is similar to that in the open ocean, while larger breakwaters induce resonating effects that are observed in the straight coast problem. In fact, this is what intuition would suggest. Now let us turn to the primary objective of this investigation: How well a breakwater of reasonable finite width is capable of reproducing performance features similar to that of the straight coast when the flap is located close to the breakwater.
Figure 5a plots the variation of capture factor $C_F$ versus the wave period $T'$ for $d' = 12$ m and normal wave incidence. For equal widths of the flap and the breakwater, a strong enhancement in the performance is seen at low periods in comparison to that in the open ocean. At higher periods, however, the level of capture factor is less than that in the open ocean, but qualitatively it maintains the consistency in performance observed in the latter at high periods. As the breakwater width is enlarged, the bandwidth of high performance levels increases and, in fact, tends toward that observed in the straight coast problem. The peak of the maximum capture factor associated with the straight coast problem is almost achieved with a breakwater twice the width that of the OWSC.

Observing the trend of the performance behaviour, it can be inferred that, as the breakwater width is increased even further, the device response would resemble more closely the straight coast variation.

**Oblique Waves**

The dynamics of the OWSC are primarily governed by the diffraction phenomenon, unlike a point absorber where radiation effects are dominant. Therefore, it is expected that its performance would decrease in oblique waves and would be nonfunctioning for grazing wave incidence ($\phi = 90^\circ$) as the excitation torque becomes zero. Previous studies have shown that the performance of the OWSC decreases almost as a function of $\cos^2 \phi$ in oblique waves (Whittaker et al., 2007). However, the OWSC is a shallow-water WEC where the waves are directional. Therefore, it is expected to be located in regions with predominantly normal wave incidence or instead can be oriented in the most predominant direction. However, since the problem addressed here describes a modification to the original WEC configuration, a few computations are performed for near-normal incidence to assess the possibility of any drastic changes in the behaviour. Figure 5 shows the variation in the capture factor $C_F$ for three different angles of incidence. Qualitatively, the variation in the performance features remains the same but with the magnitudes in capture factor dropping. From the comparison of the rate of decrease in the peaks of the capture factor curve, an OWSC with a breakwater is marginally more sensitive to directional wave incidence than that in the open ocean case.

**RANDOM SEAS**

The behaviour of the system in random seas is investigated here. The analysis is performed for six of the ten total sea states that were identified by Aquamarine Power Ltd. to represent the annual wave climate at the European Marine Energy Centre wave site (Clabby et al., 2013), and a methodology presented in Sarkar, Renzi, and Dias (2013b) is adopted. Computation for the four sea states that are not performed here corresponds to high sea states, in which the spectral components from large periods that cause excitation of body resonance of the flap are not negligible and may lead to significant error in the results (Sarkar, Renzi, and Dias, 2013b). Also, the exploitable wave energy as a percentage of the total wave energy incident would be significantly reduced in higher sea states compared to that in low seas because of restrictions in the power take-off system, which act as a kind of performance inhibitor (see Whittaker and Folley, 2012). It is worth mentioning that the occurrence of the higher sea states (which are not analysed here) in the annual wave climate representation is much lower than that the occurrence of the higher sea states, which are not analysed here, in the annual wave climate representation is much lower than that of the investigated here. Therefore, the analysis would give a fair picture although slightly incomplete.

The Bretschneider spectrum for fully developed wind waves (Goda, 1999) has been used here and is given by the form:

$$S(f) = 0.257H_{1/3}^2T_{1/3}^{-4}f^{-5}\exp[-1.03(T_{1/3}f)^{-4}]$$  \hspace{1cm} (28)

where $S$ is the spectral density, $H_{1/3}$ is the significant wave height, $T_{1/3}$ is the significant wave period, and $f$ is the frequency. The six different sea states considered in this analysis are described in Table 1.

Figure 6 plots the capture factor for the six sea states considered. A breakwater of the same width as that of a flap captures more power than that of a single flap in the open ocean with the exception of sea number 6. However, if the mentioned sea states were to represent the complete wave climate for a particular location, the annual energy production would then be considerably higher.
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Table 1 Representation of six sea states

<table>
<thead>
<tr>
<th>Sea number</th>
<th>$H_{1/3}$ (m)</th>
<th>$T_{1/3}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>6.1</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>6.1</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>7.9</td>
</tr>
<tr>
<td>4</td>
<td>1.9</td>
<td>9.0</td>
</tr>
<tr>
<td>5</td>
<td>1.1</td>
<td>9.8</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Fig. 6 Comparison of the capture factor $C_F$ of the OWSC in random seas, located at a distance $d'$ of 12 m from the breakwater for various widths of the breakwater and six different seas

for equal widths of both structures. For a breakwater of width $b = 1.5w$, a higher capture factor is observed across all the seas analysed, with $C_F > 2 \times (C_F)_{\text{Open Ocean}}$ for the first three seas. For the breakwater of width twice that of the flap, a capture factor of at least twice that in the open ocean across all the seas is observed. In fact, for sea number 3, the capture factor is almost three times that in the open ocean. The random sea analysis gives an insight into the behaviour of such systems in realistic sea conditions. The representation of the wave climate is strongly site-specific. However, the breakwater definitely can be used as a performance enhancer for the OWSC at locations where the occurrence of low seas is significantly predominant.

In comparison to the hybrid system presented in Schoolderman et al. (2011), the three-dimensional dynamics of the OWSC-breakwater system enable it to have much higher levels of efficiency than the former. Also, in the present system, the breakwater is utilised to enhance the performance of a well-known WEC, and as such, design modifications are not required.

CONCLUSIONS

The possibility of using a breakwater to enhance the performance of an OWSC is explored in this paper. A three-dimensional mathematical model is presented in the framework of linear potential theory to analyse the hydrodynamics and performance of the OWSC in such circumstances. The results show that when the OWSC is located very close to the breakwater, it is possible to significantly enhance the performance, with the absorption bandwidth depending on the width of the breakwater. The general behaviour tends toward that observed in the case of a straight coast when the breakwater width is enlarged.

As far as the hydrodynamic variations are concerned, spikes in the radiation parameters $\mu'$ (added inertia) and $\nu'$ (radiation damping) are observed near $k'd' = (m + 1/2)\pi$, $m = 1, 2, \ldots$, which are similar to the resonance observed in the straight coast problem. The excitation torque on the OWSC tends toward zero for integral values of $k'd'/\pi$ as the breakwater width is enlarged, which is explained by a standing wave phenomenon in the straight coast problem. The random sea analysis reveals that this kind of system would be most effective for locations with a higher occurrence of low sea states. From the perspective of economics, a fairly good enhancement in the performance of the OWSC can be obtained with a breakwater one and a half times the width of the OWSC.

In a realistic situation, a breakwater can span a long distance, and an array of OWSCs can be implemented in such a situation. It would be interesting to understand the effects of the mutual interactions between the OWSCs in the presence of the breakwater. Such a topic could be a future research direction for this work.

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REFERENCES


The 2D Green’s function:

\[ G_x(x, y; \xi, \eta) = \frac{1}{4\pi} H_0^1(\sqrt{(\xi - x)^2 + (\eta - y)^2}) \]

satisfies the Helmholtz equation:

\[ (\nabla^2 + k^2)G_x = 0, \quad G_x = \frac{1}{2\pi} \ln r \quad \text{as} \quad r \to 0 \]

where \( r = \sqrt{(\xi - x)^2 + (\eta - y)^2} \). Application of Green’s integral theorem to \( \varphi_x \) and \( G_x \) for the whole fluid domain yields:

\[ \varphi_x(x, y) = \frac{1}{2\pi} \int_{\Omega} \frac{\nabla \varphi_x \cdot \nabla G_x}{\nabla G_x} \, dA + \int_{\partial \Omega} \left( \frac{\partial \varphi_x}{\partial n} - \frac{\partial \varphi_x}{\partial n} G_x \right) \, ds \]

where \( \nabla \varphi_x = \varphi_x(0, -e, y) = \varphi_x(0, e, y) \) and \( \nabla \varphi_x = \varphi_x(-e, e, y) - \varphi_x(e, -e, y) \) denote the modal potential difference across the two sides of flap and breakerwater, respectively. Application of the 2D spatial potential on the kinematic boundary conditions on the flaps gives:

\[ \int_{\Omega} \frac{\nabla \varphi_x}{\nabla G_x} \cdot \nabla G_x \, dA = \frac{1}{2\pi} \int_{\Omega} \nabla \varphi_x \cdot \nabla G_x \, dA + \int_{\partial \Omega} \left( \frac{\partial \varphi_x}{\partial n} - \frac{\partial \varphi_x}{\partial n} G_x \right) \, ds \]

and on the breakerwater, it gives:

\[ \int_{\Omega} \frac{\nabla \varphi_x}{\nabla G_x} \cdot \nabla G_x \, dA = \frac{1}{2\pi} \int_{\Omega} \nabla \varphi_x \cdot \nabla G_x \, dA + \int_{\partial \Omega} \left( \frac{\partial \varphi_x}{\partial n} - \frac{\partial \varphi_x}{\partial n} G_x \right) \, ds \]

Making the following change of variables:

\[ u_1 = \frac{2\eta}{w}, \quad u_2 = \frac{2\eta}{b}, \quad v_1 = \frac{2y}{w}, \quad v_2 = \frac{2y}{b} \]

and

\[ P_{a1}(u_1) \quad Q_{a1}(u_1) \quad P_{a2}(u_2) \quad Q_{a2}(u_2) \]

\[ P_{a1}(u_1) \quad Q_{a1}(u_1) \quad P_{a2}(u_2) \quad Q_{a2}(u_2) \]

\[ \varphi_x(x, y) = \frac{1}{2\pi} \int_{\Omega} \frac{\nabla \varphi_x}{\nabla G_x} \cdot \nabla G_x \, dA + \int_{\partial \Omega} \left( \frac{\partial \varphi_x}{\partial n} - \frac{\partial \varphi_x}{\partial n} G_x \right) \, ds \]

\[ \int_{\Omega} \frac{\nabla \varphi_x}{\nabla G_x} \cdot \nabla G_x \, dA = \frac{1}{2\pi} \int_{\Omega} \nabla \varphi_x \cdot \nabla G_x \, dA + \int_{\partial \Omega} \left( \frac{\partial \varphi_x}{\partial n} - \frac{\partial \varphi_x}{\partial n} G_x \right) \, ds \]

\[ \int_{\Omega} \frac{\nabla \varphi_x}{\nabla G_x} \cdot \nabla G_x \, dA = \frac{1}{2\pi} \int_{\Omega} \nabla \varphi_x \cdot \nabla G_x \, dA + \int_{\partial \Omega} \left( \frac{\partial \varphi_x}{\partial n} - \frac{\partial \varphi_x}{\partial n} G_x \right) \, ds \]
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\[ D_{p2} = -\frac{i \pi \kappa_w b}{2} \int_{-1}^{1} (1 - u^2)^{1/2} U_p(u_2) \frac{d}{d^2 + (v_1 w - b u_2)^2} \]
\[ \times \left[ \kappa_H H^{(1)}_4 ((\kappa_H/2)^2 / (4d^2 + (v_1 w - b u_2)^2) \right] \]
\[ - 2H^{(1)}_4 ((\kappa_H/2)^2 / (4d^2 + (v_2 b - w u_2)^2) \right] \]
\[ \times \left( v_1 w - b u_2 \right)^2 \]
\[ D_{p1} = -\frac{i \pi \kappa_w b}{2} \int_{-1}^{1} (1 - u^2)^{1/2} U_p(u_1) \frac{d}{d^2 + (v_2 b - w u_1)^2} \]
\[ \times \left[ \kappa_H H^{(1)}_4 ((\kappa_H/2)^2 / (4d^2 + (v_2 b - w u_1)^2) \right] \]
\[ - 2H^{(1)}_4 ((\kappa_H/2)^2 / (4d^2 + (v_2 b - w u_1)^2) \right] \]
\[ \times \left( v_2 b - w u_1 \right)^2 \]

The two systems of equations, Eqs. 42 and 43, are finally solved by using a numerical collocation scheme as outlined in previous studies (see, e.g., Renzi and Dias, 2012, 2013).