Lift of a Rotating Circular Cylinder in Unsteady Flows

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A cylinder rotating in steady current experiences a lift known as the Magnus effect. In the present study, the effect of waves on the Magnus effect has been investigated. This situation is experienced with the novel, floating offshore vertical axis wind turbine (VAWT) concept called the DEEPWIND concept, which incorporates a rotating spar buoy and thereby utilizes seawater as a roller bearing. The a priori assumption and the results suggest that the lift in waves, to a first approximation, may be represented by a formulation similar to the well-known Morison formulation. The force coefficients are experimentally found to depend primarily on the ratio between the surface speed of the cylinder and the outer flow velocity.

INTRODUCTION

The DEEPWIND project is a collaborative research project on future deep-sea wind turbine technologies, which is partially funded by the European Commission (EC) through the 7th Framework Programme (FP7). The DEEPWIND concept was described in Vita et al. (2009) and is illustrated in Fig. 1. It consists of a VAWT (i.e. Darrieus rotor) as the energy-capturing device rigidly fixed on top of a long upright tube connected to the seabed with mooring lines. The submerged part of the tube acts as a rotating spar buoy and utilizes seawater as a roller bearing instead of having a main bearing between the rotor and the support structure. This solution solves some of the structural problems experienced by VAWT in the past. A correct description of all forces on the rotating spar buoy is paramount in order to be able to simulate the proposed wind turbine concept and to assess the performance.

The flow around the rotating spar buoy may be broken down into two classical topics within hydrodynamics: (1) flow around a circular cylinder in current and waves, and (2) circular cylinder with rotation in steady current. The flow around a circular cylinder in current and waves exerts a resultant in-line force (drag) and cross-flow force (lift) on the cylinder (Sumer and Fredsøe, 1997). Cylinder diameter, surface roughness and inclination all influence the resultant force, as well as current speed and wave motion. A rotating cylinder in steady current experiences an additional force perpendicular to the motion (lift) known as the Magnus effect (Batchelor, 1967; Hoerner and Borst, 1975). Furthermore, the wall shear stress on the cylinder surface exerts a resultant friction opposite the cylinder rotation. Theodorsen and Kegier (1945) give a detailed account of revolving cylinders in stagnant fluid.

Although the flow around the rotating spar buoy may be broken down into classical topics within hydrodynamics, little is known about the combined effect of current, wave and rotation. The observed behaviour of the inertia coefficient (in-line force) of a horizontal cylinder beneath waves was attributed (Chaplin, 1984; see Sumer and Fredsøe (1997) for a review) to the steady, recirculating streaming that builds up around the cylinder and is disrupted by the formation of separation vortices in the wake. Rotating cylinders have also been considered as wave energy converters (Chaplin and Retzler, 1996). Chaplin and Retzler (1996) used linear potential theory to determine the Magnus lift force of a deeply submerged horizontal rotating cylinder beneath waves. It is seen, although expressed as a torque acting on the wave energy device, that the Magnus lift force equates to the Kutta-Joukowski theorem with a time-dependent velocity. By extension of the theorem of Blasius, this solution is actually found as a special case of a cylinder moving in an unbounded ideal fluid (Milne-Thomson, 1962). The Kutta-Joukowski theorem, known from steady flow around a rotating body, relates the lift force to the density of the fluid times the circulation times the speed of the fluid, \( F_L = \rho \Gamma U \). However, experimental results with rotating cylinder in steady currents (Hoerner and Borst, 1975) have found the lift to be dependent of viscosity (or the Reynolds number).

The present study addresses the combined effect of wave, current and rotation through an extensive experimental campaign concerning lift of a rotating cylinder in pure oscillatory flow. The
experimental results are analysed and supported by numerical simulations to address possible scaling effects.

**EXPERIMENTAL SETUP**

The experiments were carried out in a 35-m long, 3-m wide and 1-m deep current flume with a carriage running on rails on top of the flume. The water depth of the flume was maintained at 0.7 m, and the model spar buoy, a section of a circular cylinder, was mounted under the carriage, as illustrated in Fig. 2. Current could be directed through the flume, while waves were simulated by an oscillatory movement of the carriage. The global coordinate system \((X, Y, Z)\) was arranged so \(X\) was the stream-wise direction, \(Z\) was vertical, positive upwards, and \(Y\) was in the transverse direction according to the right-hand rule.

The current velocity was measured with an ultrasonic current meter (Minilab SD-12), while the carriage position along the length of the flume was measured with a laser distance-measuring device (SICK DT50-P2113). During post-processing, the position signal was differentiated with time to give the velocity of the carriage, \(U = dX/dt\).

Figure 3 shows a close-up of the model. The cylinder, which had a diameter \(D = 160\) mm, was split in three parts. The middle part, which had a length of 0.4 m, was the measuring section, while the bottom and top parts were dummy sections. The two dummy sections were rigidly fixed to a drive shaft, while the measuring section was suspended in two 2-component force gauges (DHI-205/30). One force gauge was placed inside the bottom dummy section and the other was placed inside the top dummy section. One end of each of the two force gauges was rigidly fixed to the drive shaft, while the other end of the force gauges held the measuring section. A small gap between the top and bottom cap of the measuring section allowed for slight movement of the measuring section relative to the drive shaft so that the displacement could be measured by the strain gauges on the force gauges. The measuring section was made rough by gluing sand, with a mean grain size \(d_{so} = 0.3\) mm, onto the cylinder surface. A 5-cm band at the bottom of the top dummy section and at the top of the bottom dummy section was similarly made rough. The relative roughness expressed with the equivalent sand roughness was estimated to be \(k_{r}/D = 4 \cdot 10^{-3}\).

The cylinder drive shaft was mounted through ball bearings to an adjustable support frame, which allowed adjustment of the vertical position of the cylinder in the water column. The support frame consisted of two box profiles (placed on top of the carriage), four M20 threaded steel rods and two 200 mm \(\times\) 400 mm \(\times\) 15 mm steel plates assembled with nuts and washers. The ball bearings holding the drive shaft were positioned in the centre of each of the steel plates.

The motor controlling the cylinder rotation was placed next to the cylinder drive shaft between the two steel plates of the support frame. The motor gearing was adjusted to give a rotational speed of the motor drive shaft between 20 rpm and 110 rpm depending on the voltage supplied to the motor. The gearing between the motor drive shaft and the cylinder drive shaft was 32/32 = 1, i.e. the angular position of the motor is identical to the angular position of the cylinder (rotation around the vertical axis, \(Z\)). This allowed the angular position of the cylinder and the rotational speed to be measured at the motor drive shaft with two continuous potentiometers (Contelec WAL305). During post-processing, the two outputs were combined to achieve a 360° measurement range, and a crossing-analysis of the position signal was used to determine the rotational speed.

The output from the two force gauges, \((F_{x1}; F_{y1})\) and \((F_{x2}; F_{y2})\), respectively given in the local coordinate system rotating with the cylinder, was re-calculated during post-processing to the global coordinate system (Figs. 2 and 4) using the angular position of the circular cylinder, \(\psi\).

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} = R_{z} \begin{bmatrix}
F_{x1} + F_{x2} \\
F_{y1} + F_{y2}
\end{bmatrix} = \begin{bmatrix}
\cos(\psi) & -\sin(\psi) \\
\sin(\psi) & \cos(\psi)
\end{bmatrix} \begin{bmatrix}
F_{x1} \\
F_{y1}
\end{bmatrix} + \begin{bmatrix}
F_{x2} \\
F_{y2}
\end{bmatrix}
\]

(1)

**Oscillatory flow**

\[
U = U_{m} \sin(2\pi v/T)
\]

**Fig. 2** Schematic drawing of model submerged rotating cylindrical cylinder mounted via ball bearing and support frame to the carriage

**Fig. 3** Submerged rotating circular cylinder

**Fig. 4** Definition sketch for oscillatory flow with cylinder rotation
where \( F_x \) is the resulting in-line force, \( F_y \) is the cross-flow (lift) force and \( R_z \) is the rotation matrix.

**NUMERICAL MODEL**

The numerical model solving the incompressible Reynolds-averaged Navier-Stokes equations (Eq. 2) used the OpenFOAM toolbox version 1.7.1 (http://www.openfoam.org):

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \tau_{ij} \right] + F_{bi} \tag{2}
\]

combined with the local continuity equation:

\[
\frac{\partial u_i}{\partial x_i} = 0 \tag{3}
\]

where \( u_i \) are the mean (phase-resolved) velocities, \( x_i \) are the Cartesian coordinates, \( p \) is the pressure, \( \nu \) is the kinematic viscosity, \( \tau_{ij} \) is the Reynolds stress tensor and \( F_{bi} \) represents body forces used to drive the flow (see below). To close the system of equation, the SST \( k-\omega \) turbulence model was used with standard tuning coefficients (Menter, 1994). The numerical model is essentially similar to that described and validated by Fuhrman et al. (2009).

The above equations are solved in two dimensions, subject to the following boundary conditions: The cylinder surface is considered as a friction wall with no-slip boundary conditions, i.e. all velocity variables are set to zero or to the rotational speed at the surface of the cylinder. Velocities at the inlet and outlet are imposed a zero gradient condition at the cylinder wall and a fixed total pressure at the inlet and outlet boundary. Within these, the computational cells are stretched with a progression of 1.03 in the radial direction to provide adequate resolution of the boundary layer forming on the cylinder.

The flow is driven with an oscillatory body force, \( F_B \), in the \( X \)-direction (\( i = 1 \)):

\[
F_B = F_{B0} + \sum_{n=1}^{N} \frac{\partial U_n}{\partial t}
\]

where \( U \) is the free-stream velocity defined in Eq. 5, and \( F_{B0} \) (\( = 0 \)) is a global forcing term allowing any mean flux.

The CFD simulations were performed in 2D. From flow around non-rotating circular cylinders, it is known that the vortex shedding in the turbulent wake regime occurs in cells along the length of the cylinder. Therefore, the maximum resultant force acting on the cylinder over the length of a single cell (as calculated here) may be larger than the force acting on the cylinder over its total length. However, for small Keulegan-Carpenter numbers, the correlation length of the vortex shedding will be several multiples of the cylinder diameter (Sumer and Fredsøe, 1997).

**TEST CONDITIONS**

Table 1 and Table 2 summarize the test conditions for pure oscillatory flow experiments. Note that \( U_m \) in the tables is the amplitude of the free-stream velocity defined by:

\[
U = U_m \sin(2\pi ft + \varphi)
\]

where \( f = 1/T \) is the wave frequency (\( T \) is the period of the oscillatory motion), and \( \varphi \) is the phase lag. The amplitude of the orbital motion is calculated from:

\[
a = \frac{U_m T}{2\pi}
\]

<table>
<thead>
<tr>
<th>( U_m ) (m/s)</th>
<th>( T ) (s)</th>
<th>( a ) (m)</th>
<th>( KC )</th>
<th>( Re )</th>
<th>( \omega/2\pi )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>1.74</td>
<td>0.07</td>
<td>2.2</td>
<td>3.2 ( \times 10^4 )</td>
<td>30~95</td>
<td>6</td>
</tr>
<tr>
<td>0.23</td>
<td>3.00</td>
<td>0.16</td>
<td>4.3</td>
<td>3.6 ( \times 10^4 )</td>
<td>30~77</td>
<td>4</td>
</tr>
<tr>
<td>0.26</td>
<td>2.14</td>
<td>0.14</td>
<td>3.4</td>
<td>4.1 ( \times 10^4 )</td>
<td>29~75</td>
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</tr>
<tr>
<td>0.28</td>
<td>2.47</td>
<td>0.19</td>
<td>4.3</td>
<td>4.4 ( \times 10^4 )</td>
<td>29~74</td>
<td>5</td>
</tr>
<tr>
<td>0.32</td>
<td>2.14</td>
<td>0.22</td>
<td>4.3</td>
<td>5.1 ( \times 10^4 )</td>
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<td>1</td>
</tr>
<tr>
<td>0.39</td>
<td>1.74</td>
<td>0.26</td>
<td>4.2</td>
<td>6.2 ( \times 10^4 )</td>
<td>19~71</td>
<td>5</td>
</tr>
<tr>
<td>0.44</td>
<td>4.55</td>
<td>0.32</td>
<td>12.5</td>
<td>7.0 ( \times 10^4 )</td>
<td>64</td>
<td>1</td>
</tr>
<tr>
<td>0.44</td>
<td>5.15</td>
<td>0.36</td>
<td>14.1</td>
<td>7.0 ( \times 10^4 )</td>
<td>22~78</td>
<td>4</td>
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<tr>
<td>0.44</td>
<td>5.76</td>
<td>0.40</td>
<td>15.8</td>
<td>7.0 ( \times 10^4 )</td>
<td>62</td>
<td>1</td>
</tr>
<tr>
<td>0.44</td>
<td>6.36</td>
<td>0.44</td>
<td>17.4</td>
<td>7.0 ( \times 10^4 )</td>
<td>22~76</td>
<td>4</td>
</tr>
<tr>
<td>0.44</td>
<td>6.97</td>
<td>0.49</td>
<td>19.1</td>
<td>7.0 ( \times 10^4 )</td>
<td>62</td>
<td>1</td>
</tr>
<tr>
<td>0.44</td>
<td>7.57</td>
<td>0.53</td>
<td>20.8</td>
<td>7.0 ( \times 10^4 )</td>
<td>30~76</td>
<td>4</td>
</tr>
<tr>
<td>0.44</td>
<td>8.20</td>
<td>0.57</td>
<td>22.4</td>
<td>7.0 ( \times 10^4 )</td>
<td>62</td>
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<td>0.61</td>
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<td>0.46</td>
<td>6.2</td>
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<td>19~69</td>
<td>5</td>
</tr>
<tr>
<td>0.50</td>
<td>4.02</td>
<td>0.98</td>
<td>12.4</td>
<td>7.9 ( \times 10^4 )</td>
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<td>0.53</td>
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<td>0.69</td>
<td>8.2</td>
<td>8.5 ( \times 10^4 )</td>
<td>26~69</td>
<td>5</td>
</tr>
<tr>
<td>0.56</td>
<td>4.02</td>
<td>1.25</td>
<td>14.0</td>
<td>8.9 ( \times 10^4 )</td>
<td>62</td>
<td>1</td>
</tr>
<tr>
<td>0.62</td>
<td>4.02</td>
<td>1.54</td>
<td>15.6</td>
<td>9.9 ( \times 10^4 )</td>
<td>60</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1 Test conditions, physical model experiments: pure oscillatory flow; cylinder diameter \( D = 160 \) mm. \( N \) is the number of tests performed in the range reported for the cylinder rotation, \( \omega/2\pi \).
Lift of a Rotating Circular Cylinder in Unsteady Flows

Table 2 Test conditions, numerical modelling: pure oscillatory

<table>
<thead>
<tr>
<th>$U_m$ (m/s)</th>
<th>$T$ (s)</th>
<th>$a$ (m)</th>
<th>$KC$ (-)</th>
<th>$Re$ (-)</th>
<th>$\omega/2\pi$ (rpm)</th>
<th>$N$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.14</td>
<td>5.5</td>
<td>1.0</td>
<td>1.0</td>
<td>6.8·10^6</td>
<td>15, 20</td>
<td>2</td>
</tr>
<tr>
<td>1.61</td>
<td>7.8</td>
<td>2.0</td>
<td>2.1</td>
<td>9.6·10^6</td>
<td>15, 30</td>
<td>2</td>
</tr>
<tr>
<td>1.97</td>
<td>9.6</td>
<td>3.0</td>
<td>3.1</td>
<td>1.2·10^7</td>
<td>15, 30</td>
<td>2</td>
</tr>
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<td>2.28</td>
<td>11.0</td>
<td>4.0</td>
<td>4.1</td>
<td>1.4·10^7</td>
<td>0~15</td>
<td>4</td>
</tr>
<tr>
<td>2.47</td>
<td>18.5</td>
<td>7.4</td>
<td>7.6</td>
<td>1.5·10^7</td>
<td>10</td>
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<td>2.28</td>
<td>22.0</td>
<td>8.0</td>
<td>8.3</td>
<td>1.4·10^7</td>
<td>15</td>
<td>1</td>
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<td>2.66</td>
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<td>8.8</td>
<td>1.6·10^7</td>
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<td>1</td>
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<td>3.38</td>
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<td>13.4</td>
<td>13.8</td>
<td>2.0·10^7</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

The Keulegan-Carpenter number and the Reynolds number are defined as:

$$KC = \frac{2\pi a}{D} = \frac{U_m T}{D}$$  \hspace{1cm} (7)

$$Re = \frac{\alpha U_m}{\nu} = \frac{T U_m^2}{2\pi \nu}$$  \hspace{1cm} (8)

respectively, where $\nu$ is the kinematic viscosity (0.01 cm$^2$ s$^{-1}$). The water temperature was 20 °C.

The cylinder rotation may be expressed as the ratio of surface speed and the free-stream velocity (the speed ratio):

$$\alpha = \frac{\omega D}{2U_m} = \frac{\omega R}{U_m}$$  \hspace{1cm} (9)

where $R$ is the cylinder radius and $\omega$ is the angular frequency of the cylinder. In Table 1 and Table 2, the range of angular frequency tested has been indicated.

A number of validation tests were performed for the physical model experiments with steady current: The current speed was in the range $U_c = 0.06$~$0.35$ m/s, and the angular frequency of the cylinder was in the range $\omega/2\pi = 20$~$80$ rpm. The Reynolds number for the current, defined as:

$$Re_c = \frac{DU_c}{\nu}$$  \hspace{1cm} (10)

was in the range $1$·$10^4$~$6$·$10^4$, and the speed ratio, $\omega R/U_c$, was between 0 and 5.

RESULTS

Steady Current

For clockwise rotating circular cylinder in steady flow (see sketch in Fig. 6), the velocity of the upper surface of the cylinder is in the same direction as the free-stream velocity. Separation is thereby delayed on the upper surface, whereas it occurs earlier on the lower surface. As a result, the pressure distribution on the cylinder surface is altered when the rotation is present. Pressure is reduced on the upper surface and increased on the lower surface, causing a positive net lift force. This effect is known as the Magnus effect. Rotation in the opposite direction reverses this effect and causes a negative lift force.

![Fig. 6 Lift and drag of a rotating cylinder as a function of speed ratio: Magnus force. Circles and squares: present data; lines: Hoerner and Borst (1975), data band of $C_L$ attributed to Reynolds number.](image)

The non-dimensional mean lift force (or lift coefficient) is defined by:

$$C_L = \frac{F_L}{\frac{1}{2} \rho DU_c^2}$$  \hspace{1cm} (11)

and the mean drag coefficient, $C_D$, is defined in the same way based on the in-line force, $F_D$. Figure 6 shows the measured lift and drag coefficient as a function of the speed ratio, $\omega R/U_c$. Experimentally measured lift and drag coefficient for Reynolds numbers between 40,000 and 660,000, reported by Hoerner and Borst (1975), has also been included in Fig. 6. The figure shows a good comparison between the present data and the data reported by Hoerner and Borst (1975). The discrepancy of lift coefficients at higher speed ratios is expected to be related to differences in Reynolds number and cylinder roughness, as detailed below.

The data band reported by Hoerner and Borst (1975) for the lift coefficient is attributed to the range of Reynolds numbers. Specifically, the negative lift is reported to occur for small speed ratios in the transition (or critical) Reynolds number range of $10^5$ to $5$·$10^5$. The present results are obtained for a rough cylinder. Cylinder roughness has the effect of reducing the transition Reynolds number range (Achenbach and Heinecke, 1981). Because of the reported transition Reynolds number range and the lack of information about roughness conditions, it is concluded that the data reported by Hoerner and Borst (1975) are for smooth cylinders. This may explain why the present results indicate a negative lift for small speed ratios although the current Reynolds numbers are smaller than those listed by Hoerner and Borst (1975). Note that the highest current Reynolds number in the present experiments will be associated with the smaller speed ratios. For subcritical Reynolds numbers, the drag coefficient takes the value obtained in the case of smooth cylinders irrespective of cylinder roughness (Achenbach and Heinecke, 1981). Generally, $Re_c$ is in the subcritical regime, and a good comparison between the present data and the drag coefficient reported by Hoerner and Borst (1975) is observed.
Potential flow theory predicts zero drag force \((C_D = 0)\) and a lift force given by:

\[
F_y = \rho U^2 \frac{\partial U}{\partial t} = \rho (2\pi \omega R^2)U,
\]

the Kutta-Joukowski theorem. Hence, potential flow theory predicts a lift coefficient given by:

\[
C_L = \frac{\rho (2\pi \omega R^2)U}{\rho RU_y^2} = \frac{2\pi \omega R}{U_y} = 2\pi \alpha
\]

This theoretical value is much higher than experimental results suggest (Fig. 6), a discrepancy that is primarily due to viscosity. However, the measured variation of \(C_L\) with respect to the speed ratio is to some extent in accordance with the theoretically predicted linear variation – at least over a certain range of speed ratios.

**Oscillatory Flow**

The forces on a rotating cylinder moving in an ideal fluid in the positive direction of the X-axis are given by (Milne-Thomson, 1962):

\[
F_x = \rho \pi R^3 \frac{\partial U}{\partial t} = \rho A \frac{\partial U}{\partial t}
\]

\[
F_y = \rho U^2 = \rho (2\pi \omega R^2)U = \rho A 2\omega U
\]

where \(A = \pi R^2\). The in-line force is the hydrodynamic mass force of the Morison formulation, while the cross-flow force expresses the Kutta-Joukowski theorem.

Figure 7 shows the measured ensemble-averaged cross-flow force at two different \(KC\) numbers: \(KC = 4.3\) and \(KC = 14.0\). The two examples are representative of the situations \(KC < 10\) and \(KC > 10\), respectively.

Also included in Fig. 7 are the measured velocity and acceleration. The graphs show that the cross-flow force variation is proportional to the velocity for \(KC < 10\), as the Kutta-Joukowski theorem suggests, while \(F_y\) is proportional to the velocity squared, \(U^2\), for \(KC > 10\). Particularly for \(KC < 10\), there is a noticeable phase shift between the velocity and the cross-flow force. The phase shift is most clearly seen as the phase difference in the zero-crossing between the cross-flow force and velocity. An explanation for this phenomenon may be that the inertia component of the pressure around the cylinder (Sumer and Fredsøe, 1997) has a non-zero contribution to the cross-flow force. The non-zero contribution from the inertia component to the cross-flow force would suggest an asymmetry between the upper part of the cylinder and the lower part of the cylinder originating from viscosity effects in combination with the circulation or boundary layer separation. Following this line of thought, an equation for the cross-flow force may therefore be formulated as:

\[
F_y = \rho AC_1 \omega U + \rho AC_m \frac{\partial U}{\partial t}
\]

where the coefficient, \(C_1\), has been introduced to handle frictional effects, i.e. in potential flow \(C_1 = 2\) and \(C_m = 0\).

Expressing the Kutta-Joukowski theorem, the first term in Eq. 16, as in the steady flow around a rotating cylinder:

\[
\rho AC_1 \omega U \equiv \frac{1}{2} \rho C_D U |U|
\]

gives another formulation for the cross-flow force, namely:

\[
F_y = \frac{1}{2} \rho C_D U |U| + \rho AC_m \frac{\partial U}{\partial t}
\]

Note that the velocity-squared term in Eq. 11 is written in the form \(U |U|\) to ensure that the force is always in the direction of the velocity. From Eq. 17, it is found that \(C_F\) can be described by:

\[
C_F = \frac{C_L}{\frac{U}{\pi \omega R}} \quad \text{or} \quad C_L = \pi C_F \frac{\omega R}{|U|}
\]

In the steady flow around a rotating cylinder, it is not important which formulation for the cross-flow force is used, because the velocity is a constant. In oscillatory flow, it is, however, important which formulation is used if \(C_F\) and \(C_L\) are defined as constants over one wave period.

The formulation with \(C_F\) (Eq. 16) assumes that the cross-flow force is a function of the velocity, \(U\), with a small correction for the hydrodynamic mass, while the formulation with \(C_L\) (Eq. 18) assumes that the cross-flow force is a function of the velocity squared, \(U |U|\). Figure 7, as previously mentioned, shows which formulation should be used considering that \(KC = 4.3\) is representative of the flow regime \(KC < 10\) and \(KC = 14.0\) is representative of \(KC > 10\). Table 1 shows that there are zero tests with a \(KC\) number in the range 8.2 < \(KC < 12.5\). The change from one flow regime to the other may therefore occur anywhere in the range 8.2 < \(KC < 12.5\), and the limit \(KC \approx 10\) between the two regimes should therefore be regarded as a best guess based on the present data. Interestingly, the \(KC\) number range 8.2 < \(KC < 12.5\) coincides with the transverse vortex-street regime (7 < \(KC < 13\)) and the start of vortex shedding (\(KC > 7\)) in pure oscillatory flow around a non-rotating cylinder. Vortex shedding is therefore the
most likely cause for the change in flow regime in oscillatory flow around a rotating cylinder, as will be further substantiated below.

A total of 58 tests were performed with oscillatory flow around a rotating cylinder. For each test, the lift coefficient, $C_L$ (or the Kutta-Joukowski coefficient, $C_k$), and the hydrodynamic mass coefficient, $C_m$, have been determined using the method of least squares:

For $KC < 10$, the formulation with $C_k$ (Eq. 16) has been used. Figure 8 shows the lift, expressed with $C_k$ and $C_m$, as a function of the speed ratio. $C_k$ is in the order of 1 when $\omega R/U_m = 0.5$. It gradually decreases with increasing speed ratio and attains the value 0.5 for $\omega R/U_m = 4$, a value that is substantially smaller than the potential flow solution, $C_k \equiv 2$. Even at small speed ratios, such as $\omega R/U_m = 0.5$, the measured $C_k$ is only half of the value predicted by the potential flow solution. For $\omega R/U_m < 0.5$, the Kutta-Joukowski coefficient is expected to increase with decreasing speed ratio, although with an upper limit given by the potential flow solution, $C_k \equiv 2$. The inertia coefficient, $C_m$, appears to attain a constant value of 0.2 over the entire range of speed ratios tested ($0.5 < \omega R/U_m < 4$), if the scatter in the data points is neglected. Figure 7a includes the predicted cross-flow force. The predicted cross-flow force is calculated from the determined force coefficients using Eq. 16. The figure shows a good agreement between the measured and predicted cross-flow force. There are, however, some soft fluctuations in the measured cross-flow force that are not captured by the proposed formulation. If such fluctuations are located around the phase of maximum acceleration ($\omega t = 0$ and $\omega t = 180$), it may have a large impact on the determined inertia coefficient and may explain the scatter in the data point observed for $C_m$.

For $KC > 10$, the formulation with $C_L$ (Eq. 18) has been used. Figure 9 shows the lift as a function of the speed ratio. The number of data points with $KC > 10$ are limited. However, the lift coefficient, $C_L$, shows a clear trend to increase with the speed ratio, while the inertia coefficient again scatters around the value 0.2.

The Kutta-Joukowski coefficient, $C_k$ (Fig. 8), has been recalculated to a lift coefficient, $C_L$, using Eq. 19. The corresponding lift coefficient is shown in Fig. 10. The lift coefficient is, as for $KC > 10$, a function of the speed ratio. Indeed, the data points for $KC < 10$ and $KC > 10$ coincide (this may, however, not be the case for larger speed ratios). From Fig. 10, it is seen that the lift coefficient $C_L \approx 4$, at a speed ratio $\omega R/U_m = 2$, for oscillatory flow around a rotating cylinder. This is the same value as for steady flow around a rotating cylinder. For $\omega R/U_m < 2$, the lift coefficient in the unsteady case is higher than in the steady case. And for $\omega R/U_m > 2$, the lift coefficient is smaller in the unsteady case than in the steady case. In the steady case for an ideal fluid, $\omega R/U_c = 2$ corresponds to the instance when the two stagnation points coincide to form a single stagnation point at the bottom of the cylinder surface. Above this value, the single stagnation point will move off into the fluid as either a single stagnation point or two stagnation points (Batchelor, 1967). This may possibly be related to the observed difference in lift coefficient between the unsteady and steady case.

SCALING TO FULL-SCALE

The expected dimensions of the full-scale rotating spar buoy relevant to this study can be summarized as: a diameter in the order of 6 m to 8 m, a rotational speed in the order of 10 rpm to 15 rpm, a roll/pitch of less than 15 degrees, and a draft in the order of 200 m. The rotating spar buoy is considered to be subject...
to waves with a relationship between wave height and wave period given by:

\[ T = B_0 \cdot \sqrt{H/g} = B_1 \cdot \sqrt{H} = B_1 \cdot \sqrt{2ae^{-2\pi z/L}} \]  

(20)

where \( B_1 = B_0 g^{-1/2} \) is assumed to be between 3.3 and 4.0 m\(^{-1/2}\) s\(^1\) \((10.3 < B_0 < 12.5)\), and \( g \) is the gravitational acceleration. These two values correspond to the relationship found in the northern North Sea between the significant wave height and the peak wave period, and between the significant wave height and the mean wave period, respectively. The wave height in Eq. 20 has been converted to amplitude, \( a \), of the orbital motion using deep-water airy wave theory, where \( L \) is the wave length and \( z \) is the vertical coordinate positive upwards measured from the free surface. Equation 20 illustrates that a large set of wave periods and amplitudes should, as Table 1 shows, have been considered to cover the full-scale conditions.

Geometrical similarity between model and full-scale gives a model scale of 1:37.5 to 1:50. Dynamic similarity between physical model experiments and full-scale may only be attainable at full-scale because of mutually exclusive scaling laws. The physical model experiments were scaled using Froude scaling. Froude scaling, together with geometrical similarity, allowed reproduction of the \( KC \) number, \( \omega R/U_m \) and \( k/D \) in the model in accordance with the expected full-scale values. This, however, means that the Reynolds number was not scaled correctly in the physical model experiments. The consequence of this is considered to be small because surface roughness and the high rotational speed suggest a turbulent boundary layer flow (Theodorsen and Kegier, 1945; Achenbach and Heinecke, 1981; Sarpkaya, 1976). Therefore, the force coefficient presented in the previous section is considered representative of full-scale conditions.

The lift of rotating circular cylinder in oscillatory flow was, in parallel with the physical model experiments, calculated at full-scale using the numerical model presented above. At full-scale, the numerical tests help translate the laboratory results to real-life scale. The numerical tests were in the range \( KC < 14 \) (Table 2). For each test, the force coefficients were determined as for the physical model experiments, i.e. the Kutta-Joukowski and inertia coefficients were determined for \( KC < 10 \). Subsequently, \( C_L \) was re-calculated to \( C_L \).

![Fig. 11 Lift, expressed with \( C_L \) and \( C_m \), of a rotating cylinder in oscillatory flow as a function of the speed ratio (1 < \( KC < 14 \)). Circles, triangles: numerical tests; lines: best-fit to physical model results.](image)

Figure 11 shows the results of the numerical tests. Also included is the best-fit to the results of physical model experiments. The figure shows that the numerical results are in accordance with the experimental results. However, the numerical model predicts a lift coefficient in the order of 15 to 20\% larger than the best-fit to the experimental results; this factor may be taken into account when scaling to full-scale. The somewhat larger discrepancy of lift coefficients at \( \omega R/U_m > 5 \) is expected to be related to the scatter of data points. Given a larger data set, it is expected to disappear.

Figure 12 presents a comparison between the ensemble-averaged cross-flow force measured in the experiments and simulated with CFD at the same \( KC \) numbers and speed ratio as in Fig. 7. The comparison is good, although the simulated lift coefficient is larger than the measured lift coefficient for these two cases. Both the experiments and the CFD simulations show that the cross-flow force variation is proportional to the velocity for \( KC < 10 \), as the Kutta-Joukowski theorem suggests, while \( F_c \) has a higher harmonic for \( KC > 10 \). In the previous section, it was made probable that the change in regime was due to vortex shedding. Indeed, the CFD simulations show that vortex shedding occurs for \( KC \) larger than approximately 10 and not for small \( KC \) numbers, as illustrated by the flow patterns presented in Figs. 13 and 14. Inspection of the force time series simulated by CFD reveals that \( KC \sim 10 \) is representative of the change in flow regime. However, the process by which the regime change takes place might look similar to that presented by Justesen (1991) for oscillatory flow around a non-rotating cylinder (see also Sumer and Fredsøe (1997)). The limited analysis of vortex shedding patterns and regimes presented herein has been undertaken to point out that this aspect of the problem would be worth exploring further.

The CFD simulations predicted a lift coefficient, \( C_L \), of about 4.8 and 3.0, respectively, while \( C_L \) in the experiment was measured to be about 4.6 and 2.9, respectively. The lift coefficient predicted by the CFD simulations is thus approximately 6\% larger than that predicted by the experiments for the cases shown.

![Fig. 12 Ensemble-averaged cross-flow force measured in the experiments and simulated with CFD at different \( KC \) numbers: (a) \( KC \sim 4.2 \), and (b) \( KC \sim 13.9 \).](image)
CONCLUSIONS

The Magnus effect has been experimentally determined in oscillatory flow around a rotating cylinder.

Oscillatory flow around a rotating cylinder is explored in connection with the DEEPWIND concept for floating offshore wind. The size of this floating vertical axis wind turbine (VAWT) implies that $KC < 10$ covers the expected wave motion relative to the rotating spar buoy.

For $KC < 10$, the cross-flow force (lift) coefficient, $C_L$, was termed the Kutta-Joukowski coefficient to indicate that the cross-flow force variation in time is primarily a function of the velocity, as the potential flow solution through the Kutta-Joukowski theorem predicts. $C_L$ is in the order of 1 when $\omega R/U_m = 0.5$. It gradually decreases with increasing speed ratio and attains the value 0.5 for $\omega R/U_m = 4$, a value that, however, is substantially smaller than the potential flow solution, $C_L = 2$.

For $KC > 10$, the cross-flow force variation in time has been found to be proportional to the velocity squared, $U_t/U$, and it has been found that it, to a first approximation, may be represented by a formulation similar to the well-known Morison formulation. The lift coefficients, $C_L$, have been found to depend primarily on the ratio between the surface speed of the cylinder and the outer flow velocity, increasing with increasing speed ratio.

Scaling to full-scale has been addressed theoretically, assisted by CFD modeling, and it is found that the experimental results are representative of full-scale conditions.

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Two aspects have at present been neglected: The elliptical orbital motion of water particles and the roll/pitch angle. The total in-line force on a vertical cylinder is practically uninfluenced by the orbital motion, unless the ellipticity of the motion is quite large ($U_0/V_0 > 0.7$ to 0.8), according to Sumer and Fredsøe (1997). Deep-water waves have an ellipticity in the order of 1. If the in-line force is affected by the ellipticity, then cross-flow force may possibly be impacted. The roll/pitch angle is small, and it is assumed that the independence principle applies for $KC < 8$ (Sumer and Fredsøe, 1997). In the range $8 < KC < 20$, the in-line force is influenced even for small roll/pitch angles (Sumer and Fredsøe, 1997), owing to the disruption of the transverse vortex street. The transverse vortex street or the beginning of vortex shedding was, in the present study, found to influence the cross-flow force. The angle of attack due to the roll/pitch angle may therefore possibly influence the cross-flow force in the range $8 < KC < 20$. 

Fig. 13 Streamlines and pressure distribution ($c_p = (p - p_\infty)/(\frac{1}{2} \rho U_0^2)$) simulated by CFD at $KC = 4.2$, $\alpha = 1.9$ and $t = T/4$ ($2\pi ft + \varphi = 90$ deg)

Fig. 14 Streamlines and pressure distribution ($c_p = (p - p_\infty)/(\frac{1}{2} \rho U_0^2)$) simulated by CFD at $KC = 13.8$, $\alpha = 0.9$ and $t = T/4$ ($2\pi ft + \varphi = 90$ deg)


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