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Simulation of Bichromatic Second-Order Stokes Waves in a Numerical Wave Flume

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ABSTRACT

The generation and propagation of second-order Stokes waves in a semi-infinite, narrow channel of uniform depth is simulated numerically. The wave motion is produced by the small-amplitude, bichromatic, oscillatory motion of a generic planar wavemaker. Both the first- and second-order problems are solved by a boundary element approach. A novel second-order radiation boundary condition, previously developed for monochromatic waves, is extended herein to deal with a bichromatic wave system. The suitability of the second-order radiation boundary condition is verified by monitoring the wave profile and through energy considerations on the computational domain. The accuracy of the computed results is checked by comparing the time-domain results with published frequency-domain solutions and laboratory data. It is concluded that the present approach provides an accurate and efficient technique to simulate the generation and propagation of second-order bichromatic Stokes waves in a two-dimensional wave flume.

INTRODUCTION

The proper interpretation of the experimental results obtained in a laboratory wave flume requires an appreciation of the limitations of the experimental setup and of the characteristics of the wave field produced; this is especially true if the waves are of finite-amplitude. The present paper considers the nonlinear wave field produced in a laboratory flume consisting of a long, relatively narrow channel with a planar wavemaker at one end and an absorbing beach at the other. Such facilities are common worldwide for the study of wave-structure interactions. Although the generated wave field is two-dimensional, this restriction is unimportant in many applications, and the use of a single planar wavemaker to produce the wave motion results in a high degree of control over the generated wave field.

In order to accurately predict the nonlinear wave field produced by planar wavemaker motions, several investigators have studied the problem using the Stokes expansion procedure in the frequency-domain. Hudspeth and Sulisz (1991) presented a complete second-order solution to the problem for both flap- and piston-type wavemakers by an eigenfunction expansion approach. Monochromatic wavemaker motions were assumed and a set of complementary solutions was developed to satisfy the nonhomogeneous boundary conditions on the wavemaker and free-surface. This solution has been extended to account for bichromatic wavemaker motions by Moubayed and Williams (1993) and, more recently, by Schaffer (1996). The frequency-domain solution yields the linear (first-order) and quadratic (second-order) transfer functions of wave elevation, from which the time history of the free-surface to second-order may be obtained.

An alternative approach to seeking a second-order solution in the frequency domain is to solve the wavemaker problem directly to this order in the time-domain. This technique has been successfully applied to analyze several wave-structure interaction problems in both 2 and 3 dimensions by Isaacson and his co-workers (Isaacson and Cheung, 1990, 1991; Cheung and Isaacson, 1991; Isaacson and Ng, 1993; Ng and Isaacson, 1993) and was recently applied to the wavemaker problem by Zhang and Williams (1996). The analysis of Zhang and Williams (1996), however, was restricted to the study of monochromatic wavemaker motions. One feature of this latter analysis was the development and evaluation of a new second-order radiation boundary condition to be applied at the far-field boundary of the computational domain. The accuracy of this radiation boundary condition was verified by considering the forms of the computed second-order wave profiles and by monitoring the difference between the energy flux at the far-field boundary and the power input by the wavemaker. Relative errors of less than 2% were obtained in all test cases examined.

The present paper extends the application of the second-order radiation condition developed by Zhang and Williams (1996) to enable the generation and propagation of bichromatic second-order Stokes waves to be simulated. Potential flow theory is assumed and the initial/boundary value problems for the velocity potentials at both first- and second-order are solved by a high-order, boundary element approach. The bichromatic second-order potential is decomposed into free and forced wave components, and each of these consists of 4 separate terms corresponding to the double frequency, sum-frequency, and difference frequency of the two constituent wave components. The previously developed second-order radiation boundary condition is modified to take into account the asymptotic properties of these bichromatic wave components. The suitability of the second-order radiation boundary condition formulation and the correctness of its implementation are verified by monitoring the simulated wave profile, and through energy considerations on the computational domain. The accuracy of the computed results is also checked against published second-order frequency-domain solutions and laboratory data for bichromatic waves. It is concluded that the present approach provides an accurate and efficient technique to simulate the generation and propagation of bichromatic second-order Stokes waves in a laboratory wavetank, and it may provide a useful tool to analyze the results of experiments carried out in these type of facilities.

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