

# Direct Spectral Fatigue Analysis Method for Offshore Structures

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## ABSTRACT

**A direct spectral analysis method is used for the fatigue analysis of an offshore jacket structure subjected to random wave excitation. Hence the stress range power spectral density function matrix is evaluated in a form that can be calculated directly and conveniently, without the evaluation of the input force spectral matrix required by a full spectral analysis. The results should be identical to those of the full spectral method, if both methods retain the same number of modes, and differ from the approximate ones of the reduced spectral method.**

## INTRODUCTION

Welded offshore structures are subject to cyclic loading that may lead to the development and propagation of cracks. Fatigue analysis is primarily addressed to design of the welded joints between jacket members as these are the weakest points for fatigue. Cronin et al. (1979) summarized the three kinds of spectral fatigue analysis as follows.

The first kind of spectral fatigue analysis based on deterministic analysis is the Discrete Wave Analysis Method, which is customarily used in fatigue analysis and life estimates for offshore structures. Its advantages are that it is simple and convenient for calculating fatigue life, and its disadvantage is that it is difficult to include dynamic effects in a realistic way.

The second kind of spectral fatigue analysis recognizes the probabilistic nature of the real sea state and of the dynamic response of offshore structures under wave loads. Its normal form is the Full Spectral Method, which includes a very complicated evaluation of the force spectral density matrix caused by the wave forces due to the velocity and acceleration of the water particles. Hence it is difficult to use in engineering practice. For multi-degree of freedom (multi-DOF) 3-dimensional structural systems, large computer storage and long CPU times are needed to carry out the hot-spot-stress range spectral density of joints. Cronin pointed out that "the large number of nodes in real structures necessitates further simplification of the structural model." This not only restricts the application but also reduces the use of this approach in engineering.

The third kind of spectral fatigue analysis is based on the assumption of a linear relationship between the wave height and the stress range of response; it yields the Reduced Spectral Method, which is simple and practical but whose results are approximate.

Zheng and Cheng (1985) suggested a Direct Spectral Analysis Method for stochastic response analysis of offshore structures in which the difficulty of calculating the force spectrum of the sys-

tem is avoided. This approach was extended to nonclassically damped systems by Zheng (1990) and then Zheng and Cheng (1992) combined it with the perturbation method to give the changes of the natural frequencies and response caused by structural modifications.

In the present paper this method is further extended to the fatigue analysis for offshore structures. The spectral density of the stress range is calculated directly from the wave height spectral density function exactly as in the full spectral method, but the calculation of the force spectral density matrix required by Borgman (1965) is avoided. Combining this method with the modal synthesis technique of Zheng and Cheng (1986, 1988), the fixed interface method is usually employed to assemble the upper structure including the surrounding water, and the lower structure including piles inserted into the seabed. The huge number of DOF of the system when analysed in a discrete form by using the finite element method (FEM) can be reduced by taking less of the fixed interface normal modes. This greatly reduces the CPU time required.

A numerical example is presented to verify that the suggested method is suitable for engineering application and so clearly has advantages when used in design.

## DIRECT SPECTRAL ANALYSIS METHOD

### Governing Equation

In configuration space, the Lagrange governing equation of damped  $n$  DOF discrete linear systems obtained by FEM can be written in terms of the mass matrix  $\mathbf{m}$ , damping matrix  $\mathbf{c}$  and stiffness matrix  $\mathbf{k}$  as:

$$\mathbf{m}\ddot{\mathbf{x}} + \mathbf{c}\dot{\mathbf{x}} + \mathbf{k}\mathbf{x} = \mathbf{f}(t) \quad (1)$$

where  $\mathbf{f}(t)$  is the force vector of stationary random excitations. From the theory of random vibration, the relationship between the output  $\mathbf{S}_{xx}$  and input  $\mathbf{S}_{ff}$  power spectral density function matrices is given via a transfer function matrix  $\mathbf{H}_x(\omega)$  as:

$$\mathbf{S}_{xx}(\omega) = \mathbf{H}_x(\omega)\mathbf{S}_{ff}(\omega)\mathbf{H}_x^{*T}(\omega) \quad (2)$$

where the asterisk denotes complex conjugate and  $\mathbf{H}_x(\omega)$ ,

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