

Dynamic Curvature in Catenary Risers at the Touch Down Point Region: An Experimental Study and the Analytical Boundary-Layer Solution

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ABSTRACT

Recently, the dynamics of catenary risers, in the absence of shock conditions against the soil, has been studied in the vicinity of the touch down point (TDP) by means of asymptotic methods and boundary-layer theory (Aranha, Martins and Pesce, 1997). The major outcome of this previous study is a simple mathematical expression that enables one to relate, in time and space, the dynamic curvature and TDP excursion when an oscillatory tension is applied to the riser. Such a dynamic asymptotic boundary-layer solution, though simple, contains all the nonlinear features that result from the nonlinear geometric boundary condition at the soil, of the one-side type. In order to validate the boundary-layer solution, an experimental study has been conducted with a structural model, at the IPT laboratory facilities. The riser model was instrumented along its length by an array of strain-gages, of the resistive type, with static and periodic dynamic problems being investigated in detail. Comparison with the analytical asymptotic solution has been carried out showing outstanding agreement. Not only time-average, rms and maximum amplitude variation of curvature over the riser length are recovered, but also the corresponding curvature time-histories along the riser's span, within the boundary layer, fully validating the analytical approximate solution.

INTRODUCTION

Recent research on the technical feasibility of the steel catenary riser alternative applied to the oil industry (Phifer et al., 1994) gave rise to a number of questions concerning the local dynamics in the touch down point (TDP) region. The precise evaluation of dynamic curvature in this region is mandatory, as fatigue life calculation is a crucial point in design. Driven by waves and by the floating unit, the riser undergoes oscillatory motions, causing TDP position to vary in time. In a purely two-dimensional problem, we can think of an instantaneous TDP position in close analogy to the instantaneous center of rotation in the kinematics of a rolling cylinder. Physically, the oscillatory variation of the TDP makes the riser lift off and lie down cyclically from and on the sea floor, causing large variations of curvature in this region. As the riser lies down on the floor repeatedly, the curvature, and thus the bending stresses, vary from a very small value (exactly null if the soil is supposed to be rigid) to a somewhat relatively large one, on the order of the maximum static catenary curvature. As is well-known, catenary curvature is discontinuous at TDP, since only the tangency boundary condition is satisfied, the null curvature condition being lost. This condition can only be recovered by introducing the bending stiffness effect.

In fact, as flexural rigidity is rather small, considering the length of the riser, bending stiffness plays a minor role in the

global dynamics, that is, dominated by the geometric (catenary) rigidity. However, close to the ends, particularly to the TDP, the flexural rigidity effect is important. This is an example of the so-called beam-string problem, a singular perturbation problem, being generally and properly treated under the boundary-layer technique approach (Love, 1927, art. 273^A; Kevorkian and Cole, 1981, art. 2.8, page 97).

The locally two-dimensional dynamic (in the absence of shock against the soil — a subcritical regime) problem can then be addressed by a boundary-layer type technique, giving rise to an analytical asymptotic (nonlinear) solution for the curvature problem in the TDP region (Aranha, Martins and Pesce, 1997). As a matter of fact, besides being simple in form, this analytical solution depends solely on the dynamic tension and on the instantaneous TDP displacement, which can be easily evaluated from the catenary solution. (We refer to the equations of a heavy inextensible cable under the action of steady current and dynamic loads due to waves and floating unit motions.) The local nonlinearities are accounted for by the TDP oscillatory motion, and the curvature is written as a nonlinear time-function of tension and TDP displacement. Making use of this boundary-layer solution, a number of qualitative and quantitative results could be obtained, as those previously presented by Pesce and Pinto (1996). In this way it has been possible, for example, to analytically determine the maximum curvature variation, which is crucial in the fatigue analysis, making explicit the main parameters that control such variation, and may be useful in the design of a catenary riser.

Such an analytical approach, being simple in form, enables one to treat catenary riser problems not only from an analytical point of view but also from a design-oriented one. Notice that design-oriented approaches, based on classical methods of applied mathematics, have been rare in literature. A good example can be found in Bernitsas et al. (1985), cited by Patel (1992) in his

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