International Journal of Offshore and Polar Engineering
Vol. 7, No. 4, December 1997 (ISSN 1053-5381)

Application of Higher Order Transfer Functions in Modeling Nonlinear Dynamic Bahavior of Offshore Structures

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ABSTRACT

The nonlinear behavior of a single degree of freedom systems (SDOF) subjected to wave and current is studied through dynamic analysis and application of higher order transfer functions. The dynamic analysis shows that nonlinear responses appear at the resonance frequencies of the SDOF systems even when no wave energy exists at those frequencies. Higher order transfer function models based on Volterra series are used to model the nonlinear responses and represent them successfully either as quadratic or cubic dominant. A brief discussion on the application to multidegree of freedom systems is made.

INTRODUCTION

Flexible offshore structures operating in deep water can undergo an appreciable amount of nonlinear dynamic behavior at their natural frequencies under severe wave loading conditions, even though these natural frequencies are well outside the dominant wave frequency range. A number of studies has been conducted on modeling the nonlinear components in the behavior of flexible structures. S. B. Kim et al. (1989) used a higher order transfer function model based on Volterra series and successfully estimated quadratic transfer functions to model the low-frequency nonlinear motions of a tension leg platform (TLP) obtained from scaled experimental data. Donley and Spanos (1990) expanded previous statistical linearization methods and proposed a statistical quadratization method to estimate the low-frequency TLP surge response that could not be properly modeled with the linearization technique. In their study Morison's equation was used to predict the wave forces, and the calculations were performed in the original position of the TLP with the addition of current.

The present study attempts to evaluate the relative importance of various nonlinear effects on the response of flexible structures, identifying the order of nonlinearity. Before embarking on the study of more complicated and realistic structures, a SDOF system was used to calculate the dynamic response to forces obtained from Morison's equation accounting for two different nonlinear effects. The nonlinear effects considered are the drag term in Morison's equation, with and without current, and the calculation of the wave forces at each instant of time in the displaced position of the structures. The nonlinear transfer functions were obtained from the given wave input and the computed response as output. From the coherency spectrum of the transfer function the degree of nonlinearity of each of the nonlinear effects was determined. The application to a standing cylinder modeled as a multidegree of freedom system and to a TLP is briefly discussed.

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Received February 22, 1997: revised manuscript received by the editors July 17, 1997. The original version was presented directly to the Journal.

KEY WORDS: Dynamic structural analysis, offshore structures, irregular waves, Volterra series, nonlinear transfer functions.

FORMULATION

Equilibrium Equation for a SDOF

The SDOF used in this work consists of a circular disk of diameter D with very small thickness t and a spring with stiffness k which attaches the disk to the bottom. The disk has a mass $mt = \rho_s \pi D^2 t/4$ where ρ_s is the equivalent mass density of the disk.

Calling y the disk displacement, the equation of motion can be written as:

$$mt\ddot{y} + ky = C_M \rho_w \frac{\pi D^2}{4} a_x t - C_m \frac{\pi D^2}{4} \rho_w t\ddot{y} + \frac{1}{2} C_D \rho_w D t |u - \dot{y}|(u - \dot{y}) \qquad (1)$$

where $\ddot{y}$ and $\dot{y}$ are the cylinder acceleration and velocity, respectively; C_M = inertia coefficient; $C_m = C_M - 1$ = added mass coefficient; C_D = drag coefficient; ρ_w = water density and, a_x and u are the water particle acceleration and velocity, respectively. The force terms on the right hand side of the equation are from the modified Morison's equation in which the relative movement of the flexible cylinder and the water particle is taken into account (Chakrabarti, 1987). The first term on the right-hand side is the inertia force from fluid motion only, and the second term is the added mass term due to the movement of the cylinder in the water. The last term is the drag force term caused primarily by flow separation downstream from the cylinder, and it depends on the relative velocity between water particles and the cylinder.

To obtain the response y in Eq. 1, the constant average acceleration method was used in this study with a step-by-step numerical integration (Clough and Penzien, 1990).

Nonlinear Transfer Function Model

The output $y(t)$ of a nonlinear system can often be expressed by a functional power series of the input $x(t)$ as defined by Volterra (1959). The discrete form of this representation in the frequency domain is:

$$\begin{aligned} Y(f_m) &= H_1(f_m)X(f_m) + \sum_{i+j=m} H_2(f_i, f_j)X(f_i)X(f_j) \\ &\quad + \sum_{i+j+k=m} H_3(f_i, f_j, f_k)X(f_i)X(f_j)X(f_k) \\ &= Y_L(f_m) + Y_Q(f_m) + Y_C(f_m) \end{aligned} \qquad (2)$$