

# Numerical Investigation of Focused Waves on Uniform Currents

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In this paper, a 2D numerical model for the interaction between focused waves and uniform currents is developed based on the fully nonlinear potential theory. The model is developed by using a time-domain higher-order boundary element method (HOBEM) combined with a mixed Eulerian-Lagrangian technique as a time marching process. The focused wave trains are numerically generated by a piston wavemaker on a uniform current. The proposed model is verified by comparison with the published results on monochromatic wave interaction with a current, bichromatic wave interaction with a current over a plane slope, and pure focused wave groups. The influence of the uniform current on the properties of the focused waves, including the maximum focused wave crest, shift of focal position and focal time, and spectrum bandwidth, is further studied.

## INTRODUCTION

In the complicated ocean environments, there may exist some exceptionally large and steep water waves devastating offshore structures despite their low possibility of occurrence. Many works have been carried out to experimentally and numerically study the physical properties and the generation mechanisms of extreme waves (Baldock et al., 1996; Johannessen and Swan, 2001; Kharif and Pelinovsky, 2003; Liu and Hong, 2005; Borthwick et al., 2006; Ning et al., 2009). In most of the works, the method of wave focusing has been widely used to generate extreme waves. The related research also shows that the extreme waves do not arise as part of the regular wave train, but occur as individual events at one point in space and time. In the real sea, waves generally coexist with currents, and numerous works on regular wave and current interactions have been carried out (Grue and Palm, 1985; Isaacson and Cheung, 1993; Kim et al., 1998; Koo and Kim, 2007). However, most of the research on focused waves was carried out in quiescent water. Studies of the interaction between focused waves and currents are still rare (Wu and Yao, 2004; Touboul et al., 2007; Hjelmerik and Trulsen, 2009; Merkoune et al., 2013). Actually, the wave-current interaction plays an important role in the formation of extreme waves (Lavrenov and Porubov, 2006). Besides initiating an extreme wave, the current can also influence the characteristics of a formed extreme wave. Therefore, it is necessary to carry out further investigations to gain a better understanding of the interaction between extreme waves and currents.

In this paper, a 2D fully nonlinear numerical wave tank is developed based on a HOBEM and the mixed Eulerian-Lagrangian approach to update the instantaneous free surface. As a time-marching scheme, a fourth-order Runge-Kutta (RK4) method is used to update time integration combined with regridding at each time step. The incident focused waves are generated by the spatial-temporal focusing mechanism under the real-time motion of the piston wavemaker. The current effect is realized by introducing

the current speed to the fully nonlinear free surface boundary conditions. Then the developed numerical model is utilized to investigate the influence of the uniform colinear current on the focused waves. The effects of some important parameters, such as the current direction, current speed, focused wave amplitude, and wave spectrum bandwidth, are further studied. Apart from this, the shifts of the focal position and focal time due to the current are also given in the present study.

## MATHEMATICAL FORMULATIONS

A Cartesian coordinate system  $oxz$  is employed such that the origin  $o$  is in the plane of the undisturbed free surface,  $x = 0$  is at the left end of the domain, and  $z$  is positive upwards. It is assumed that the fluid is incompressible and inviscid and the flow is irrotational so that a velocity potential  $\varphi(x, z, t)$  exists and satisfies the Laplace equation inside the fluid domain  $\Omega$ :

$$\nabla^2 \varphi = 0 \quad \text{in } \Omega \quad (1)$$

For a 2D wave propagation problem with a steady uniform current parallel to the  $x$ -axis, the total velocity potential  $\varphi$  can be described as:

$$\varphi = Ux + \phi(x, z, t) \quad (2)$$

where  $U$  is the steady uniform current and  $\phi(x, z, t)$  represents the unsteady wave potential also satisfying the Laplace equation.

On the free surface  $\Gamma_f$ , the fully nonlinear kinematic and dynamic boundary conditions are satisfied on the instantaneous free surface. They are given in the Eulerian scheme as follows:

$$\left. \begin{aligned} \frac{d\mathbf{x}_s}{dt} &= \nabla\phi + U\vec{i} \\ \frac{d\phi}{dt} &= -g\eta + \frac{1}{2}\nabla\phi \cdot \nabla\phi \end{aligned} \right\} \quad \text{on } \Gamma_f \quad (3)$$

where  $\vec{i}$  represents the unit vector in the  $x$  direction,  $g$  is the acceleration due to gravity,  $\mathbf{x}_s$  denotes the position vector of a free surface particle,  $\eta$  is the instantaneous free surface profile, and  $d/dt$  is the material derivative, i.e.,  $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$ , where  $\mathbf{v}$  is the fluid particle velocity.

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