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Hydroelastic Waves Generated by Point Loads in a Current

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Unsteady hydroelastic waves generated by impulsively starting surface and submerged concentrated loads in a fluid with an underlying uniform current are studied analytically. The fluid is assumed to be homogeneous, incompressible, inviscid, and of finite depth. For the case of irrotational motion with small-amplitude deflections, linear potential-flow theory is employed. The Laplace equation is the governing equation, with the dynamic condition representing a balance among the hydrodynamic, elastic, and inertial forces and the downward applied load. It is shown that the analytical solution, obtained by the Laplace–Fourier integral transform, consists of steady-state and transient responses. For the steady response, an explicit expression is further derived by the residue theorem, while the transient response is obtained by the stationary-phase method. These expressions allow the effects of various physical parameters on the hydroelastic responses to be studied in detail. It is found that the flexural gravity wave motion depends on the ratio of current speed to phase or group speeds.

INTRODUCTION

The occurrence of a very thin but large floating structure (VLFS) in the ocean and the presence of an ice sheet in the Polar Regions can be modeled as a thin elastic plate floating on water. One fundamental problem of interest is the hydroelastic responses caused by a concentrated load, which may find application for the VLFS or ice sheet used as a roadway or an aircraft runway. The steady wave patterns generated by a moving source on a three-dimensional (3D) water–ice system were studied by Davys et al. (1985), who used asymptotic Fourier analysis. It was found that the short-wavelength flexural waves propagate ahead of, while the long gravity waves appear behind, the moving source. The time-dependent response of floating ice to a suddenly starting line load was considered by Schulkes and Sneyd (1988) to study the decaying characteristics of the transient component. The time-dependent problem was further investigated by Nugroho et al. (1999) for a point load in three dimensions. The hydroelastic wave resistance associated with a point load was elucidated by Yeung and Kim (2000). A well-known monograph by Squire et al. (1996) was devoted to the rich topic of moving loads on thin elastic plates. Lu and Dai (2008) derived asymptotic solutions for the flexural gravity waves generated by transient and oscillating loads without a forward speed. Pogorelova (2011) studied the unsteady hydroelastic behavior of a floating plate caused by the horizontal straight motion of a point mass source in an infinitely deep fluid.

All the investigations mentioned above were based on the assumption that there is no underlying current in the fluid. A preliminary study on the effect of a current on flexural gravity waves was conducted by Schulkes et al. (1987). A detailed analysis on the dispersion relation of flexural gravity waves in a current was performed by Bhattacharjee and Sahoo (2007). Neglecting the inertia effect of the thin elastic plate, Bhattacharjee and Sahoo (2008) derived the asymptotic solution for the transient waves as a result of an initial elevation and an impulse in a current. Recently, Mohanty et al. (2014) considered a combined effect of current and compressive force on time-dependent flexural gravity wave motion for the cases of single- and two-layer fluids in two dimensions. In this paper, we consider unsteady flexural gravity waves on a thin elastic plate caused by the interaction of fixed concentrated line loads and the underlying current. Two impulsively starting loading singularities are considered within the linear system; one is on the plate surface and the other is submerged in the fluid. Analytical asymptotic solutions are deduced for the wave responses.

MATHEMATICAL FORMULATION

Without loss of generality, a Cartesian coordinate system fixed on the earth is used, in which the z-axis points vertically upward while $z = 0$ represents the mean water–plate interface. Therefore, the governing equation is:

$$\nabla^2 \Phi = S_0 \delta(\mathbf{x} - \mathbf{x}_0) \mathrm{H}(t) \tag{1}$$

where $\Phi(\mathbf{x}, t; \mathbf{x}_0)$ is the velocity potential for the perturbed flow, S_0 is the constant strength of the simple fluid–mass source; $\mathbf{x}$ is an observation point, and t is the time from the start. For two-dimensional (2D) cases, $\mathbf{x} = (x, z)$ and $\mathbf{x}_0 = (0, z_0)$, respectively. For three-dimensional (3D) cases, $\mathbf{x} = (x, y, z)$ and $\mathbf{x}_0 = (0, 0, z_0)$, respectively. $\delta(\cdot)$ and $\mathrm{H}(\cdot)$ are the Dirac delta and the Heaviside step functions, respectively.

Let ρ and ρ_e be the uniform densities of the fluid and the plate, respectively. Let $D = Ed^3/[12(1-\nu^2)]$ be the flexural rigidity of the plate, where E, d, and ν are Young's modulus, the thickness, and Poisson's ratio of the plate, respectively. The mass of the plate in a unit length is denoted by $M = \rho_e d$. An underlying current of intensity U is assumed to be moving from the left to right. The linearized kinematical and dynamical conditions at $z = 0$ are given

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