

Applicability of Particle Level Set Method for Simulation of Breaking Waves

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We investigate the applicability of the Hybrid Particle Level Set (HPLS) method for the simulation of breaking waves by analyzing its performance in modeling sloshing waves and the collapse of a water column in a confined tank. It is found that commonly used restrictions that delete aggressively escaping particles also remove flow features that an unrestricted version of the HPLS captures. However, the unrestricted HPLS performs badly in flow regions where the water phase becomes thin. The cause of the reduced performance is traced to errors in the reseeded procedure that is necessary to maintain adequate resolution of the surface.

INTRODUCTION

One of the greatest challenges in the simulation of oceanographic and coastal flows is the accurate and efficient modeling of the surface physics. For the case of breaking waves, the complex geometry of the surface and the entrainment of air present extreme challenges to traditional numerical approaches. In recent years, the level set method has become a popular tool for the modeling of 2-phase fluid flows such as bubbles, ship wakes, dam breaks and pipe flow, due to its simple representation of the surface curvature and the ease of its implementation; see Osher and Fedkiw (2001) for a review. However, in its standard form, the level set method does not guarantee the overall conservation of fluid, especially in regions of high curvature. One method that has been proposed to improve its conservation properties and accuracy is the Hybrid Particle Level Set method (HPLS) of Enright et al. (2002), in which mass-less marker particles are used to better define the location of the surface. This is achieved through correction of the level set function in the vicinity of the surface based on the position of the particles.

Here, we seek to assess the advantages and disadvantages of the HPLS method in the context of the simulation of oceanic breaking waves. To this end, we compare the HPLS method against a standard high-order level set technique that utilizes 5th-order HJ-WENO and 3rd-order Runge-Kutta schemes for the calculation of spatial and temporal gradients, respectively. When using the HPLS method for 2-phase flows, Enright et al. (2003) noted that a restriction on the correction procedure was necessary to maintain a smooth surface. Thus we compare the performance of both restricted and unrestricted HPLS methods against the high-order Level Set technique. We evaluate the accuracy of our numerical results by comparison with analytical and experimental data, focusing on the sloshing of gravity waves within a confined tank and the dam break problem. Our investigation into the sloshing of gravity waves allows us to assess the ability of the different methods to propagate small-amplitude linear waves by comparison with well-defined analytical solutions. The more challenging dam break test features the collapse of a water column from one side of a rectangular tank. As the surging water rebounds off the

opposing wall, it overturns and entrains air, similar to a wave breaking over a shallow shelf. The test cases allow us to determine the conservation properties and computational efficiency of the respective level set methods as well as their ability to capture the surface physics of breaking waves.

This paper is organized into 3 key sections, discussing first the mathematical model for our free surface capturing technique and multi-phase flow. We then give details of our discretization schemes and algorithms for solving the governing equations and implementing our level set techniques. Finally we analyze the results of a number of test cases which consider the performance of our surface capturing algorithm and finish with some conclusions.

MATHEMATICAL MODEL

In this study we investigate the motion of 2 incompressible fluids and track the movement of the dividing interface (or free surface) implicitly by the Level Set Method (LSM). In LSM, a scalar quantity φ , known as the level set function, is specified throughout the domain to represent the location of grid cells relative to the surface. Here, we define φ to be a signed distance function, which measures the shortest distance from the grid cell to the free surface (i.e. $|\nabla\varphi| = 1$) and is positive in 1 fluid phase and negative in the other. The interface at which $\varphi = 0$ (zero interface) implicitly defines the location of the surface. Since the surface moves with the fluid particles, the temporal evolution of φ is given by:

$$\frac{\partial\varphi}{\partial t} + u_i \frac{\partial\varphi}{\partial x_i} = 0, \quad (1)$$

where $u_i = (u, v, w)$ is the fluid velocity, and $x_i = (x, y, z)$ is the spatial coordinate. Eq. 1 does not enforce conservation of the fluid volume or that φ will remain a signed distance function away from the surface necessitating the use of a Hybrid Particle Level Set (HPLS) technique to improve fluid conservation and a reinitialisation technique to maintain φ as a signed distance function. These techniques are described in the section detailing the Numerical Approach.

The motion of the 2 fluids is governed by the Navier-Stokes equations:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \frac{1}{\rho(\varphi)} \left(-\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \right) + f_i, \quad (2)$$

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