Derivation and New Analysis of a Hydrodynamic Model of Speed Skate Ice Friction

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This paper presents the mathematical derivation of a new model of speed skate ice friction. The model acronym is FAST, which stands for friction algorithm using speed skate thermohydrodynamics. Mutatis mutandis, it is applicable to other ice friction problems in the hydrodynamic friction regime. This paper updates and corrects an earlier publication of the model that omitted the full derivation of the lubrication equation (Penny et al., 2007). It also updates the ice hardness equation based on new measurements, and it presents a more thorough exploration of the model results and its sensitivity to the variation of physical variables.

INTRODUCTION

Kietzig et al. (2010) recently reviewed the physics of ice friction. They identified 3 regimes:

• *boundary friction*, where the slider's load is supported primarily by contacting surface asperities;

• *mixed friction*, where the load is supported jointly by the asperities and a lubricating layer; and

• *hydrodynamic friction*, where the load is supported entirely by the lubricating layer.

Penny et al. (2007) presented an earlier review of ice friction as well as a new model of ice friction based on the assumption of hydrodynamic friction. They incorporated all the aspects of the problem that had been considered by various authors up to that time. These include frictional melting and shear stress, ploughing, heat conduction into the ice and slider and squeeze flow. Prior to this paper, no one had assembled all of these factors into a single model of lubricated ice friction. Furushima (1972) and Bäurle et al. (2007) considered frictional melting and squeeze flow. Evans et al. (1976) and Oksanen and Keinonen (1982) considered frictional melting and heat conduction. Penny et al. (2007) incorporated all of these effects into a single model and verified it against existing measurements (de Koning et al., 1992). However, space limitations precluded incorporating the complete derivation of the model's lubrication equation in that paper. In the present paper, we update Penny et al. (2007) by providing a complete mathematical derivation of the model. We also correct an error in the original model, incorporate new measurements of ice hardness, and present a new sensitivity analysis of the model. However, we ignore heat conduction into the skate blade because the skate blade temperature profile depends on its thermal history, and we are unaware of any confirmatory measurements. We also ignore the freezing point depression produced by the imposed load, because it is small.

MODEL DERIVATION

An initial version of the model was published by Penny et al. (2007), but it used old values of ice hardness and wrongly determined the skate/ice contact area. It also omitted the model derivation. The present paper corrects the error and incorporates new ice hardness measurements. Below, we consider, step by step, the physics of the hydrodynamic friction problem for a long-track speed skate blade. To simplify the problem, we consider a vertical blade gliding on a smooth, horizontal surface of ice. However, in actual skating, the blade is vertical only momentarily, as the skater shifts from the outside to the inside edge during the stroke. We plan to examine the more complex inclined blade configuration in our next paper.

Contact Length

The friction force between a skate blade and the ice depends on their contact area. For the hydrodynamic friction regime, the apparent and real contact areas are the same, because any surface roughness is assumed not to fully penetrate the lubricating liquid. A long-track speed skate blade is typically around 1.1 mm wide. It is ground flat and does not have a hollow like a hockey skate blade. We assume that the contact width, w, is identical to the blade width. The contact length on the ice surface, l, can then be computed from the vertical force balance:

$$lwp = mg \tag{1}$$

where p is the hardness of the ice, m is the mass of the skater (gliding on 1 skate), and g is the acceleration of gravity. The hardness of refrigerated arena ice was recently measured by Poirier et al. (2011), and the measurements were fitted with Eq. 2:

$$p = 14.7 - 0.6T \tag{2}$$

where p is in MPa and T is the ice surface temperature in degrees Celsius. Eq. 2 is valid over a temperature range from $-1°C$ to $-15°C$, but we will use it here over a wider temperature range for exploratory purposes. For a blade width of 1.1 mm, a skater mass of 75 kg and an ice temperature of $-5°C$, the contact length is 3.75 cm. A long-track speed skate blade has a length of about