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Comparison of Moment-Based Parameter Estimation Methods for Rayleigh-Stokes Distribution of Wave Crests and Troughs

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The Rayleigh-Stokes model has been widely applied to represent the probability distribution function of crests and troughs of weakly nonlinear random processes. In this study, the parameter estimates for the 3-parameter Rayleigh-Stokes probability distribution function model are obtained from application of 2 moment-based empirical parameter estimation methods, i.e. conventional method of moments and method of linear moments. Monte Carlo simulations are utilized to compare the performance of these parameter estimation approaches in estimating the parameters of the Rayleigh-Stokes distribution, and also to evaluate the uncertainty of the extreme statistics. Additionally, the effect of sample size on the uncertainty of the model statistics is evaluated. Finally, the Rayleigh-Stokes model is utilized to estimate the probability distribution function of disturbed wave crests beneath a mini-TLP, and the model performance is evaluated.

INTRODUCTION

The Rayleigh-Stokes model is a well-known probability distribution function in the field of ocean wave mechanics and is widely utilized to estimate the probability distribution function of weakly nonlinear wave crests. The model initially developed by Tayfun (1980) for offshore wave crests was based on the assumptions that:

- Waves can be modeled as a narrow-banded random process and consequently the wave crests of the linear waves follow the Rayleigh law (Longuet-Higgins, 1952).
- Wave elevations can be approximated by 2nd-order Stokes wave theory.

In Tayfun's Rayleigh-Stokes model the distribution structure was derived analytically and the underlying model parameter was obtained from its theoretical relation with the significant wave height and mean wave period. The model structure and the estimate of model parameter were subsequently modified by other research (Arhan and Plaisted, 1981; Kriebel and Dawson 1991, 1993; Tung and Huang, 1985; Tayfun, 2006). However, the different representations of the 2nd-order Rayleigh-Stokes model converge to almost identical results for deepwater conditions. The original 1-parameter Rayleigh-Stokes model was reasonably successful in estimating the probability distribution function of extreme offshore wave crests. However, the model did not consider the effects of interaction between incident, diffracted and radiated waves, hence the original model is not appropriate for wave crests close to or beneath an offshore structure.

Following an analogous methodology, Kriebel (1993) developed a 2-parameter Rayleigh-Stokes model for probability distribution function of nonlinear wave run-up interacting with a fixed vertical column in deep water. The parameters of Kriebel's model were estimated from their relation with the significant wave height and mean period, and application of the linear diffraction theory. Kriebel's Rayleigh-Stokes model was compared with experimental data and observed to underestimate the large crests (Stansberg and Nielsen, 2001; Izadparast and Niedzwecki, 2009b, 2010). Similarly, Fedele and Arena (2005) developed the general 2-parameter Rayleigh-Stokes distribution for the crests and troughs of a 2nd-order process and derived the theoretical estimates of the model parameters for the surface displacement and fluctuating wave pressure in an undisturbed field, as well as waves in front of a rigid wall. Later, Izadparast and Niedzwecki (2009a) developed a 3-parameter Rayleigh-Stokes model for wave crests in the vicinity or beneath an offshore platform. Izadparast and Niedzwecki (2009b, 2010a) utilized the 3-parameter Rayleigh-Stokes model to estimate the probability distribution function of wave run-up over vertical columns of offshore structures. In both studies, the model parameters were estimated by empirically utilizing the method of linear moments (L-moments). It was shown that the empirically estimated Rayleigh-Stokes model has considerable success in capturing the probability distribution function of complex nonlinear random variables.

In this study, the parameter estimates for the 3-parameter Rayleigh-Stokes probability distribution function are obtained from applying the conventional method of moments (MoM). The conventional moments give more weight to the tail of the distribution and thus are more suited for use in the prediction of extremes than L-moments. However, the sample moments, especially the high-order moments, are less efficient than the corresponding sample L-moments, and the uncertainty of small sample L-moments was shown to be less (Hosking, 1990; Hosking and Wallis, 1997). In previous studies, the performance of the method of L-moments and the method of moments in estimating the parameters of various probability distribution functions is compared (e.g. Hosking et al., 1985; Hosking and Wallis, 1987; Yamaguchi, 1996; Sankarasubramanian and Srinivasan, 1999; and Najafian, 2010). Here, Monte Carlo simulations are utilized to investigate and compare the performance of both approaches in estimating the parameters of the Rayleigh-Stokes distribution model. Addition-

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