

# Dynamically Adaptive Simulation of Coastal Hydrodynamics

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**This paper introduces a coastal modeling tool based on the solution of the 2D shallow water equations on a novel adaptive Cartesian grid system. While providing dynamically adaptive solutions, the new grid system is easy to implement, and it maintains the desirable property of a structured Cartesian grid. On such a grid, the governing equations are solved using a finite volume Godunov-type scheme that is designed to ensure non-negativity of water depth for applications involving a moving shoreline over complex domain topography. The new model is validated against several analytical and numerical benchmark tests.**

## INTRODUCTION

It is estimated that more than 38% of the world's population lives within 100 km and 44% within 150 km of a coast (Stewart, 2009). Coastal zones are thus economically and politically important for most countries or regions. Hence, it is vitally important to understand the different coastal processes driven by waves and currents, and to protect or reform our coastlines. At the heart of this is to accurately model the coastal hydrodynamics that provides the driving force for other processes. This has long been an active research topic; numerous computer models have been developed and reported, and Brocchini and Dodd (2008) gave a useful review. Models based on the solution to the depth-averaged shallow water equations (SWE) represent a wide range of applications (e.g., Dodd, 1998; Hu et al., 2000; Hubbard and Dodd, 2002).

A coastal zone may cover a large area and is normally featured by complicated topographies and geometries, where very complex local hydrodynamic phenomena can develop. Thus a model should possess certain features in order to give reliable solutions to a related problem. Basically a successful shallow flow model for coastal simulation should be able to represent complex domain topographies and geometries, handle a moving shoreline and perform efficient simulation for large-scale problems. Intensive research has been carried out in all of these topics during the past few decades, including the author's recent work (Liang et al., 2010). For large-scale simulations, the prevailing way to achieve local high-resolution predictions while maintaining high computational efficiency is to use nonuniform or adaptive mesh techniques; a number of this type of models has been reported in the literature (e.g. Hubbard and Dodd, 2002; Rogers et al., 2004; George and LeVeque, 2006). This work aims to introduce a model based on a new adaptive but structured grid system and to provide an alternative to the existing coastal simulation tools.

## GOVERNING EQUATIONS

The nonlinear SWE may be used to describe long wave propagation in a relatively shallow domain. They have been widely applied in modeling coastal hydrodynamics (e.g. Dodd, 1998; Hu et al., 2000; Hubbard and Dodd, 2002). Based on the hydrostatic assumption, the SWE may be derived by integrating in depth the Reynolds averaged Navier-Stokes equations and written in a matrix form as:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} + \frac{\partial \mathbf{g}}{\partial y} = \mathbf{s} \quad (1)$$

where  $t$ ,  $x$  and  $y$  represent the time and the 2 space directions;  $\mathbf{q}$  is the vector of flow variables;  $\mathbf{f}$  and  $\mathbf{g}$  are flux vectors in the  $x$ - and  $y$ -direction; and  $\mathbf{s}$  is the vector containing source terms. Ignoring the Coriolis effects and the surface stresses, the vectors are given by:

$$\begin{aligned} \mathbf{q} &= [\eta \quad uh \quad vh]^T, \\ \mathbf{f} &= [uh \quad u^2h + g(\eta^2 - 2\eta z_b)/2 \quad uvh]^T, \\ \mathbf{g} &= [vh \quad uvh \quad v^2h + g(\eta^2 - 2\eta z_b)/2]^T, \\ \mathbf{s} &= [0 \quad -C_f u \sqrt{u^2 + v^2} - g\eta \partial z_b / \partial x \quad -C_f v \sqrt{u^2 + v^2} - g\eta \partial z_b / \partial y]^T \end{aligned}$$

where  $\eta$  is the water surface level;  $u$  and  $v$  are the depth-averaged velocity components in the  $x$ - and  $y$ -directions;  $h = \eta - z_b$  represents the water depth with  $z_b$  being the bottom level;  $g$  denotes gravity acceleration;  $C_f$  is the bed roughness coefficient and can be evaluated using  $C_f = gn^2/h^{1/3}$ ;  $n$  is the Manning coefficient; and  $-\partial z_b / \partial x$  and  $-\partial z_b / \partial y$  represent the  $x$ - and  $y$ -direction bed slopes. With the water level  $\eta$  being directly used as a flow variable, the above SWE automatically admit well-balanced solutions (Greenberg and LeRoux, 1996), that is, preserve the solution of lake at rest, for applications involving complex domain topography.

## NEW DYNAMICALLY ADAPTIVE GRID SYSTEM

### Initial Grid Generation

This adaptive grid-based simulation starts from an initial grid that is generated by following 4 simple steps:

1. The computational domain is identified and fit into a rectangle.

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