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Energy Input Amplifies Nonlinear Dynamics of Deep Water Wave Groups

Thomas A. A. Adcock and Paul H. Taylor
Department of Engineering Science, University of Oxford, Parks Road, Oxford, United Kingdom

A possible physical mechanism for the formation of freak waves on the open ocean is the localized interactions between wind and waves. Such interactions are highly complex and are currently poorly understood at the scale of an individual wave. Rather than attempt to model the detailed transfer of energy from wind to waves, we simply consider the modifications to wave group dynamics of adding energy to the system. We carried out numerical experiments on isolated wave groups using an excited version of the nonlinear Schrödinger equation. Energy input enhances any soliton-like structures relative to regular waves for unidirectional propagation. For directionally spread wave groups, energy input enhances the nonlinear changes to the shapes of focused wave groups: Groups contract in the mean wave direction and expand in the lateral direction to a significantly greater degree than observed for nonexcited wave groups.

INTRODUCTION

There is substantial evidence that large waves on the open ocean occur more frequently than predicted by standard theory. See for instance Stansell (2004), Lawton (2001), Liu (2007) and reviews by Kharif and Pelinovsky (2003) and Dysthe et al. (2008). In unidirectional seas these large waves may be accounted for by nonlinear wave-wave interactions (Baldock et al., 1996; Janssen, 2003; Onorato et al., 2006; Adcock and Taylor, 2009). However, directional spreading greatly reduces these interactions (Onorato et al., 2009; Waseda et al., 2009). Studies in directionally spread seas (Gibbs and Taylor, 2005; Socquet-Juglard et al., 2005; Gramstad and Trulsen, 2007) find little extra elevation over that expected in 2nd-order theory. Real ocean waves are directionally spread (Forristall and Ewans, 1998), so there remains an open question as to why so-called freak waves occur in sea states with typical rms directional spreading between 10° and 30°.

One possible mechanism for rogue wave development is through localised wind-wave interaction. Most analyses of extreme waves neglect this, yet some energy transfer between the wind and the largest waves is likely. The theories describing wave growth were developed by Phillips (1957) and Miles (1957), advanced by Janssen (1991), and are used for orthodox modelling of sea states over areas much larger than a few wavelengths square. At the individual wave scale it is difficult to apply these. Some attempts have been made to examine the interaction of wind with extreme waves (Touboul et al., 2006; Kharif et al., 2008; Yan and Ma, 2010a and b), but these have only considered long-crested waves. The interaction between wind and waves is extremely complicated, particularly when combined with the complexity of water wave evolution. In this paper, we seek to simplify the problem and carry out numerical experiments to determine the effect of energy input on the evolution of nonlinear wave groups. We show that energy input accentuates the nonlinearity of wave-group evolution and therefore can increase the chances of a freak or rogue wave.

We use an isolated wave group to investigate the dynamics of large events, basing its profile on the so-called NewWave, the most probable shape for a large wave in a random sea (Lindgren, 1970; Boccotti, 1983; Tromans et al., 1991). Whilst this does not account for the random nature of large waves in the open ocean, it does allow us to examine in detail some aspects of the physics involved in the formation of extreme waves, which is key to understanding freak waves.

EXCITED NONLINEAR SCHRÖDINGER EQUATION

The simplest nonlinear equation for governing the evolution of wave groups on deep water is the nonlinear Schrödinger equation (NLSE):

$$i\left(\frac{\partial u}{\partial t}\right)=\left(\frac{\omega_0}{8k_0^2}\right)\frac{\partial^2 u}{\partial x^2}-\left(\frac{\omega_0}{4k_0^2}\right)\frac{\partial^2 u}{\partial y^2}+\frac{1}{2}\omega_0 k_0^2|u|^2u \qquad (1)$$

where u is the complex wave envelope, and k_0 and ω_0 are the carrier wave's wave number and frequency, respectively, and written in a reference frame moving with the linear group velocity. Whilst this is a gross simplification of the physical problem, it does provide the starting point for investigating the nonlinear dynamics of ocean waves. Henderson et al. (1999) give a comparison between the NLSE and the full water wave equations, and Adcock (2009) gives a comparison in the context of wave group evolution. The NLSE may be nondimensionalised using the transformations $T=\omega t$, $X=2\sqrt{2}k_0x$, $Y=2k_0y$ and $U=k_0/\sqrt{2}$ to give:

$$i\left(\frac{\partial U}{\partial T}\right)=\frac{\partial^2 U}{\partial X^2}-\frac{\partial^2 U}{\partial Y^2}+|U|^2U. \qquad (2)$$

We now modify this by introducing an excitation term proportional to the magnitude of the wave envelope (Eq. 3). In doing so, we do not seek to model the complex wind-wave interaction, but merely to consider the effect of energy input on the system:

$$i\left(\frac{\partial u}{\partial t}-\alpha u\right)=\left(\frac{\omega_0}{8k_0^2}\right)\frac{\partial^2 u}{\partial x^2}-\left(\frac{\omega_0}{4k_0^2}\right)\frac{\partial^2 u}{\partial y^2}+\frac{1}{2}\omega_0 k_0^2|u|^2u \qquad (3)$$

where α is the excitation parameter. This approach has been used by Miles (1984) and Leblanc et al. (2008), and for the analogous problem of energy dissipation by Segur et al. (2005). We define the energy input per period, β, under linear evolution, which will be used in this study as it has a more obvious physical significance than α. These 2 parameters are related by:

$$\beta=e^{2\alpha t_0}-1 \qquad (4)$$

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