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Radiation Stress and Depth-dependent Drift in Surface Waves with Dissipation

Jan Erik H. Weber
Department of Geosciences, University of Oslo, Oslo, Norway

Göran Broström
Norwegian Meteorological Institute, Oslo, Norway

Kai H. Christensen
Department of Geosciences, University of Oslo, Oslo, Norway

A Lagrangian analysis of the mean drift due to dissipating surface gravity waves is performed. The waves have amplitudes that vary slowly in time and space and can be forced by a prescribed wind-stress distribution normal to the free surface. The ocean depth is constant, and the analysis is valid for arbitrary wavelength/depth ratios. The derived equations for the horizontal Lagrangian mean drift contain depth-dependent forcing terms that are proportional to the Stokes drift, expressing the external forcing of the waves. There are no depth-varying radiation stress-like terms in these equations. For waves along the x-axis (1-axis), the mean drift equations contain the forcing from a depth-independent term that is identical to the divergence of the radiation-stress tensor component S_{22} of Longuet-Higgins and Stewart (1960). The mean Lagrangian wave setup of the free surface and the mean drift solutions in a rotating ocean are given for a steady balanced flow.

INTRODUCTION

Considerable interest has developed in formulating equations for the oceanic circulation that take into account the effect of surface gravity waves. In the traditional Eulerian description, the mean wave momentum is confined between the crests and the troughs of the wave, since the fluid motion is purely periodic below the troughs. However, individual fluid particles in the waves do have a slow drift, resulting in a mean wave momentum in the wave propagation direction: the so-called Stokes drift (Stokes, 1847). This drift is inherent in the waves, and whatever causes a change in a wave property—such as amplitude, wavelength or frequency—then causes a change in the mean wave momentum. Our definition here of the mean wave momentum as represented by the Stokes drift coincides with the wave pseudomomentum defined by Andrews and McIntyre (1978). However, the total momentum in the fluid is not necessarily distributed in space or time in the same way as the waves (McIntyre, 1981).

Stokes considered waves in inviscid fluids. Applications to real fluids had to wait for Longuet-Higgins (1953), who took into account the effect of viscosity on the wave drift. This has important consequences especially for shallow-water waves, since a no-slip bottom creates secondary mean momentum that through viscous diffusion eventually changes the drift velocity in the entire fluid layer. In the case of vertical integration of the Eulerian equations from the bottom to the undulating surface, the derived volume fluxes do include the effect of waves (Phillips, 1977). For deep-water waves, the wave-induced fluxes from the traditional Eulerian approach become identical to those derived from a direct Lagrangian approach (Weber et al., 2006).

For waves with amplitudes that vary in space, a concept called radiation stress, which acts to force the mean wave-induced fluxes, appears in the vertically integrated Eulerian momentum equations (Longuet-Higgins and Stewart, 1960). Recently, much effort has been spent in transforming the Eulerian equations in various ways so as to obtain the anticipated forcing from the depth-dependent radiation-stress terms, by analogy with the traditional vertical variation of the viscous stress, the turbulent Reynolds stress, or the wave-induced Reynolds stress. Most notable in this respect are the papers by Mellor (2003, 2005).

The main objective of this paper is to investigate the vertical variation of the wave forcing terms for the mean horizontal wave-induced particle drift. To do so, we utilize a Lagrangian description of motion. We consider drift due to waves with amplitudes that vary slowly in time and space. The wind stress, through a prescribed variation normal to the free surface, may act to sustain the wave amplitude. In the following we derive equations for the total momentum of the fluid, and no explicit distinction needs to be made between wave momentum and mean flow momentum. By using a regular perturbation of the Lagrangian equations, the 1st-order solution will naturally correspond to the waves, while the 2nd-order correction to this solution may have a nonzero mean value, and then correspond to the total mean flow, including the Stokes drift. Changes in the 2nd-order momentum can in general be traced back to either of 3 distinct physical mechanisms: frictional decay, atmospheric forcing, and barotropic pressure forces. All 3 mechanisms can impact the mean drift both directly and through the waves. The present investigation is valid for surface gravity waves with arbitrary wavelength/depth ratio.

For computational simplicity, the dissipative process in the fluid, promoting amplitude decay, is modelled by a linear friction, or a Rayleigh friction. We here refer to Lamb (1932) for a discussion of Rayleigh's work on waves on a running stream. In short, the viscous force per unit mass $\nu\nabla^2\vec{v}$, where ν is the kinematic viscosity, is replaced by $-r\vec{v}$. Here r is a constant friction coefficient. This procedure has been applied to weak friction in water waves by Miles (1967), Lake et al. (1977), and more recently by Mei and Hancock (2003) and Segur et al. (2005). By replacing the Newtonian friction by a Rayleigh-type friction, we lose the steady streaming in the boundary layers, and the accompanying

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