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Wavelet and Local Directional Analysis of Ocean Waves

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While local and wavelet-based methods have mostly been applied to single time records of wave data, Donelan et al. (1996) derived and applied the Wavelet Directional Method (WDM) to small arrays of wave elevation recorders. After a short review of the Morlet wavelet transform applied to random signals, the WDM is revisited and compared to a Local Triplet Analysis (LTA) where the wavelet transform replaces the discrete Fourier transform in the directional analysis. The sampling statistics of the WDM and LTA are summarized, assuming that the wave records are weakly stationary stochastic processes. All methods show consistent results when applied to real data. Time-averaged local estimates of common wave parameters closely fit the estimates from the standard analysis. Frequency-averaged local wave directions show considerable variation and, in addition, the local directional spread tends to take a significant drop during the passage of large wave groups. The methods are illustrated using offshore directional wave data from the Ekofisk field in the North Sea.

INTRODUCTION

During the European COST Action 714, Working Group 3 decided to compile a state-of-the-art report on various methods for measuring and analyzing directional ocean wave spectra. The report had nearly 40 contributors, and its compilation was finished in 2004 (Kahma et al., 2005). The concept of a directional wave spectrum and the corresponding spectral analysis of the wave data form the core of the report. Spectral analysis has a long history of investigations of ocean wave records. It assumes that the sea and hence the wave records are statistically stationary in time, and although this assumption is reasonable in general, time-localized events such as the passage of strong wave groups and transient waves cannot be investigated in this manner.

Apart from zero-crossing time domain analysis, the simplest local wave analysis is the envelope/phase representation of a wave signal, also frequently used as the narrow-band model for ocean waves.

Nevertheless, wind wave records are broad-banded, leading to irregular behavior of the envelope and the estimated local frequency. This led N. Huang to the Empirical Mode Decomposition Method, now called the Hilbert-Huang Transformation (Huang et al., 1998). Contrary to other successful applications of the method, however, this decomposition does not seem to have any evident physical basis for ocean waves. The spectrogram, consisting of local frequency spectra computed by short Fourier transforms moving along the record, is another straightforward local analysis. Finally, wavelets are localized both in frequency and time and may be applied both for investigating local events in a signal, and for representing the signal in terms of wavelet expansions (Torrence and Gilbert, 1998; Massel, 2001). The spectrogram is closely related to the time/frequency plot of the Morlet wavelet analysis discussed below, the main difference being that the length of the spectrogram window is independent of frequency, while wavelets scale with frequency.

The techniques above are limited to single wave records, but wavelet analysis has also been extended to multidimensional signals. About 10 years ago, Donelan et al. (1996) proposed a Wavelet Directional Method (WDM) for small arrays of wave elevation recorders. This analysis was mainly based on the Morlet wavelet (Grossmann and Morlet, 1984), written as a localized wave with frequency ν and wavenumber $\mathbf{k}$. Using the phase information in the wavelet coefficients from separate data channels, the wavenumber of the dominant directional wavelet is determined for each point of the wavelet (ν, t)-representation, producing a quite detailed local analysis without any predefined dispersion relation. In addition to a full $(\mathbf{k}, \nu)$-analysis, the result may be summarized in terms of marginal distributions; wavelet spectra, expressed as functions of $\mathbf{k}$; or as wavelet directional spectra, where the directional distribution is obtained from the directions of the wavenumbers. In addition to the possibility of identifying local events, this wavelet analysis offers a unique way to utilize data from platforms in slow, irregular motion, e.g. current meters mounted on subsurface floats, or arrays mounted on buoys (Graber et al., 2000).

In this paper, the WDM is reconsidered and compared to traditional directional wave analysis. It is also suggested that a Local Triplet Analysis (LTA) may be obtained simply by letting the wavelet transform substitute the Fourier transform in the conventional analysis of triplet data. All methods show consistent results when applied to real data from the Ekofisk oilfield in the North Sea; in particular, time-averaged local estimates of the spectrum and directional parameters closely fit the standard estimates. The LTA provides a local estimate of the directional spread around the local mean wave direction, and it turns out that the spread shows a significant drop during the passage of large wave groups. Preliminary results of this analysis were presented at WAVES 2005 (Krogstad, 2005). The WDM and LTA wavelet-based methods

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