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A Constitutive Equation for Fluid-particle Flow Simulation

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A constitutive equation, developed in the framework of the averaged continuum for fluid-particle systems and already proposed in previous papers (Lalli and Di Mascio, 1997; Lalli et al., 2005), has been implemented, and some relevant test cases have been carried out. In the proposed model, the effects of fluid-particle interaction are taken into account in terms of an effective viscosity, defined by means of both Newtonian and non-Newtonian schemes, suitably matched. Actually, this rheological model can represent pure fluid, more or less concentrated suspension and fully packed sediment as well; for concentration below a proper threshold value, the mixture behaves like a Newtonian fluid, and Bingham plastic features appear for concentrations approximating maximum packing value. The numerical solution is achieved by an explicit, fractional step finite difference method. Simple 2-dimensional examples, meaningful in the framework of sediment transport problems, are shown: Bingham flow in a lid-driven square cavity; dam break; and scour below pipelines. The obtained results have been discussed in comparison with numerical and experimental data available in the literature.

INTRODUCTION

The transport of particles in a flowing current is one of the most important and least understood problems in fluid dynamics, even though it has a fundamental relevance to many practical applications. These range from erosion and sediment transport problems in coastal and river engineering up through separation of particles and dynamics of suspensions in the chemical industry. Indeed, fluid-particle interaction is a very complicated phenomenon, due to the dynamic feedback between the 2 phases; even for simple configurations, the relationship between flow features and transport of particles is not known with suitable accuracy.

These problems can be studied from 2 different viewpoints: the Lagrangian (discrete) and the Eulerian (continuous). In the former, every single particle trajectory is computed (Maxey and Riley, 1983); in the latter, fluid-particle interaction is represented by means of the averaged continuum concept (Drew and Passman, 1991; Schaflinger et al., 1990; Dong and Zhang, 1999). In principle, the Lagrangian approach is very accurate, but in practice, it can be used only for very low concentrations. In fact, a large number of particles is not easy to handle, because of the computational difficulties related to time-consuming particle tracking itself (unless passive particles are considered), and because feedback effects of particles on the background flow need to be taken into account for higher concentrations.

Concerning the Eulerian viewpoint, generally 2 kinds of approaches are proposed in the literature: One is based on the definition of 2 interpenetrating continua; the other, on the description of fluid-particle systems by means of a single fluid model.

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These 2 approaches are, in principle, entirely equivalent (Jackson, 1997). However, from the computational point of view, the single phase model is obviously more convenient (in 3-D, 4 scalar equations need to be solved, rather than the 8 in the 2-phase model). Among several papers that can be found in the literature concerning the rheology of suspensions, one could mention the classic paper by Einstein (1906), related to viscosity of dilute suspensions, and more recent works (Krieger, 1972; Happel and Brenner, 1986; Leighton and Acrivos, 1987), where the results obtained by Einstein extend to higher concentrations.

In the framework of sediment-transport applications, most of the models available in the literature are based on the governing equations for single phase flow (Ahilan and Sleath, 1987; Nadaoka and Yagi, 1990; Ribberink and Al-Salem, 1995). In these models, fluid-particle interactions as well as intergranular stresses are generally taken into account in a rather simplified way. In particular, the flow is computed in many cases by hydrostatic models (Ribberink and Al-Salem, 1995; Fraccarollo and Capart, 2002; Crnjaric-Zic et al., 2004), while sediment concentration is generally assumed to be governed by the equilibrium between settling and turbulent diffusion (Ribberink and Al-Salem, 1995). In more recent works, related to bed forms analysis (e.g., Besio et al., 2004), the turbulent flow is computed by RANS, and the sediment transport is taken into account by means of empirical correlations. Bed forms dynamics is then studied in terms of flow-induced moving boundaries by curvilinear coordinate systems.

Previous works (Lalli and Di Mascio, 1997, referred to below as LDM, and Lalli et al., 2005, referred to below as LEPV) presented a numerical method for the continuous model based on the single phase fluid approach. In this continuous model, the fluid is assumed to be Newtonian in regions of dilute suspensions, with viscosity related to concentration, while for a high amount of suspended particles the intergranular forces are represented by a superposition of hindrance effect (Newtonian model) and shear-dependent behaviour (Bingham model). Relative motion is taken into account by means of the concepts of settling velocity and