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Wave-induced Motions of a Body Floating in a 2-Layer Fluid

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With the boundary integral-equation method developed by the author, the wave-exciting forces on and wave-induced motions of a floating body in 2-layer fluids are computed and their characteristics discussed. In a 2-layer fluid, for a prescribed frequency, there can be 2 incident waves with different wavelengths (the surface-wave mode with longer wavelength and the internal-wave mode with shorter wavelength). Thus the wave-exciting forces and resulting motions are computed separately for each mode of the incident wave. Investigation focuses on how the difference in the ratio of the fluid density and the vertical position of the interface may influence the motions of a body. The effect of the density ratio appears mainly in heave, changing the resonant frequency especially when the lower-fluid density is large. The effect of the interface position can be identified in roll and sway more obviously than in heave, as a shift of the roll resonant frequency and associated rapid variation in sway.

INTRODUCTION

A sharp change in the fluid density at a certain water depth owing to variation in salinity and/or temperature may be observed in a lake, an estuary or Norwegian fjords. Another example of sharp density change is a thin layer of muddy water at the bottom of harbors or channels with relatively shallow water depth. These density changes may significantly alter the hydrodynamic characteristics of a ship, but little knowledge on this has been available up to date. In the examples mentioned above, the change in density is usually confined within a thin layer called the pycnocline, and above and below this pycnocline the fluid density is practically constant. In this case, it is possible to model the fluid as a 2-layer fluid in which a density discontinuity exists at the interface between the upper (lighter) and lower (denser) layers. The fluid in each layer may be assumed to be inviscid and incompressible with irrotational motion, because the fluid viscosity has little influence on the wave-induced motions of a ship with which the present paper is concerned.

In a 2-layer fluid with the upper layer bounded above by a free surface and below by an interface, it is known that wave generation is different from that in a single-layer fluid. As described in Lamb (1932), for a given frequency there can be incident waves with 2 different wavenumbers, both on the free surface and on the interface in the no-obstacle case. However, for the wave-body interactions in 2-layer fluids, very little work had been done until recently. Linton and McIver (1995) and Cadby and Linton (2000) treated simple bodies like a circular cylinder and a sphere submerged in either the upper or lower fluid layer, and the wave radiation and scattering were studied using a multiple expansion method. Yeung and Nguyen (1999) developed an integral equation method based on singularity distribution to solve the radiation and diffraction problems of a 3-D body in 2-layer fluids with finite depth. However, numerical computations were made only for the case where a rectangular barge is floating in the upper layer and not intersecting the interface. Recently, Sturova (2003) presented a 2-D hybrid method, combining a finite element method near a body, and analytical asymptotic expressions at a distance from the body, to satisfy the radiation condition. Ten and Kashiwagi (2004) developed a boundary integral-equation (Green function) method solving directly for the velocity potential on the wetted surface of a general body which may intersect the interface. These works cited above, however, are concerned only with hydrodynamic forces on bodies, and no results are presented of wave-induced motions of a floating body in 2-layer fluids.

In this paper, with the calculation method developed by Ten and Kashiwagi, the wave-induced motions of a body in various 2-layer fluids with finite depth are computed. Main interest is placed on how the difference in the ratio of the fluid density and the vertical position of the interface may influence the motions of a body. As an important feature in a 2-layer fluid, for a given frequency, there can be 2 different wave modes in the incident wave, which correspond to the surface-wave mode with longer wavelength and the internal-wave mode with shorter wavelength. Since the ratio of the amplitude in these incident-wave modes is not given a priori, the wave-exciting forces on and resultant motions of a body are calculated for each incident-wave mode, and the effects of the interface on the wave-induced motions are discussed separately for each incident-wave mode.

FORMULATION

A 2-D general body is considered in a 2-layer fluid with finite depth. The body may intersect the interface and is assumed to oscillate sinusoidally in response to an incident wave with circular frequency ω. Fig. 1 shows the coordinate system and the notations used in the analyses below, with the origin on the undisturbed free surface and the z-axis positive in the downward direction. The free surface, the interface, and the flat rigid bottom of the water are located at $z=0$, $z=h_1$, and $z=h\ (=h_1+h_2)$, respectively.

With the linearized potential-flow assumption, the velocity potential is introduced and defined in the form:

$$\Phi^{(m)}(x,z,t)=\mathrm{Re}\big[\phi^{(m)}(x,z)\,e^{i\omega t}\big],\quad m=1,2 \tag{1}$$

$$\phi^{(m)}(x,z)=\phi_D^{(m)}(x,z)+\sum_{j=1}^{3} i\omega X_j\,\phi_j^{(m)}(x,z) \tag{2}$$

$$\phi_D^{(m)}(x,z)=\phi_0^{(m)}(x,z)+\phi_4^{(m)}(x,z) \tag{3}$$

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